



**Tools for Active Teaching and Active Learning**

# Hands-On Math

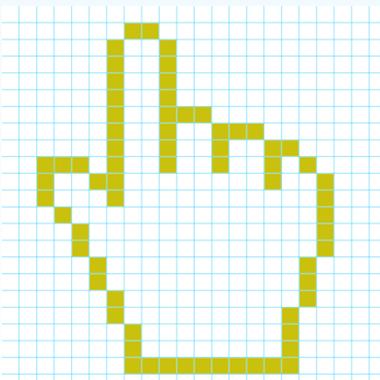


## Tangle Tables

Explore and Discover

- ✓ Addition Facts
- ✓ Multiplication Facts
- ✓ Problem Solving

## Instructor's Guide



Ventura Educational Systems

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Ben Ventura

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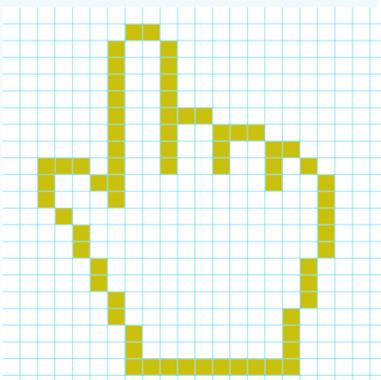
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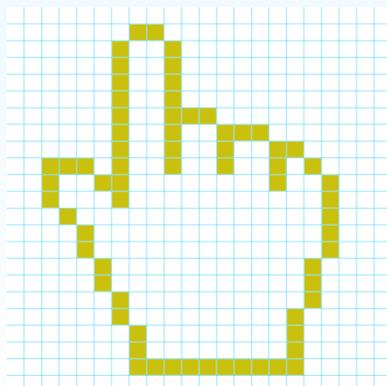
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## Overview

Tangle Tables are a great to give students an insight into the patterns inherent in the addition and multiplication tables. In elementary math education it is critically important that students see the relevance of what they are learning. Math needs to make sense. Tangle Tables helps students see the patterns of the addition and multiplication tables.

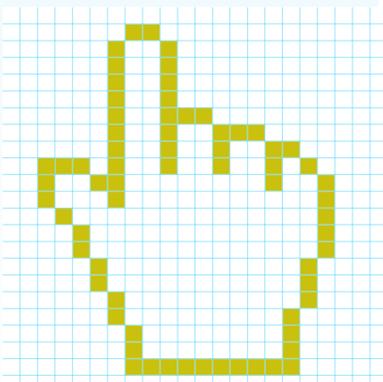
Research has shown that children learn best through active involvement in the learning process. Hands-On Math Tangle Tables is designed to be a tool that teachers can use for active teaching and active learning. Math manipulative devices can be a rich source of teaching strategies for problem solving and can be very helpful in developing an intuitive understanding of mathematical concepts. The Hands-On Math series suggests ways in which concrete learning experiences can be extended to a representational level and still remain manipulative and interactive.

This guide consists of two sections. The first part is written for the teacher and explains the functions of the app and options available. It presents ideas for instructional strategies that can be implemented with each simulated manipulative device. The second section of the manual is a set of curriculum-based activities that are designed to help the teacher in using the Hands-On Math app. These activities have been developed for elementary and middle school age children and are arranged by order of grade level where the concepts are typically introduced. Teachers will want to decide what is the best sequence for using the materials with their particular group of students. Each lesson is aimed at specific mathematical objectives including counting, representing numbers using the place value system, addition and subtraction with regrouping. Each activity is meant to be a beginning. Teachers will want to encourage the children to explore extensions of each activity with different examples. Orally discussing each activity will help to foster higher level thinking.

Hands-On Math Tangle Tables is a starting point. Learning should be fun and as students work with the app, it is my intention that they will begin discussing, sharing and creatively exploring mathematics.

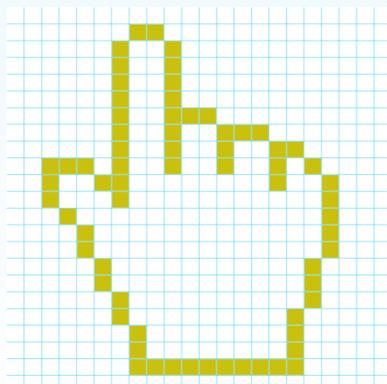
-- Fred Ventura, Ph.D.

*Visualizing mathematical concepts is the key to understanding. Models help significantly in cognitive development.*



## Introduction to Hands-On Math Tangle Tables

*Piaget's theory of cognitive development is a comprehensive theory about the nature and development of human intelligence.*



Approaches to the teaching of mathematics that rely heavily on one methodology are inherently weak and unlikely to produce optimal results. Educators have found that teaching strategies must adapt to accommodate new discoveries which expand our understanding of the learning process and new technologies which expand our delivery systems.

According to learning theory, children learn best when they are actively involved in the learning process. There are many ways to do this but one example is having children work in small groups in a laboratory/discovery situation. Small group instruction encourages variation in teaching methodology. Variation in the way in which material is presented serves the instructional process since one particular methodology may not be best for all children. Different children respond differently to a particular educational approach. The same methodology that is appropriate for one content area or developmental stage may not be appropriate in a different content area or with children who are at a different developmental stage.

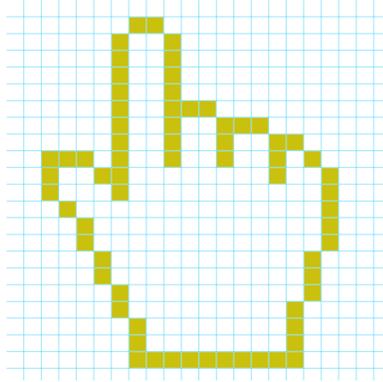
For learning mathematics an active teaching and active learning situation is a very desirable educational environment. To create it the teacher must be aware of the behavioral characteristics of the students with regard to mathematics, must be knowledgeable in the particular skills which are being taught and must be able to draw upon diverse strategies in order to decide which is the most appropriate for fostering the development of the targeted mathematical concepts.

In general, educational psychologists believe that the ability of children to learn passes through developmental stages. Each stage is characterized by particular behaviors. In the early stages learning is tied to perceptual responses. As the child matures, abstract reasoning becomes possible and concrete models are useful for laying the conceptual groundwork for new ideas, but once a concept has been internalized the concrete models are no longer necessary. The work of Swiss psychologist, Jean Piaget, has contributed a great deal to support this theory, and to foster the development of educational strategies which are consistent with the theory.



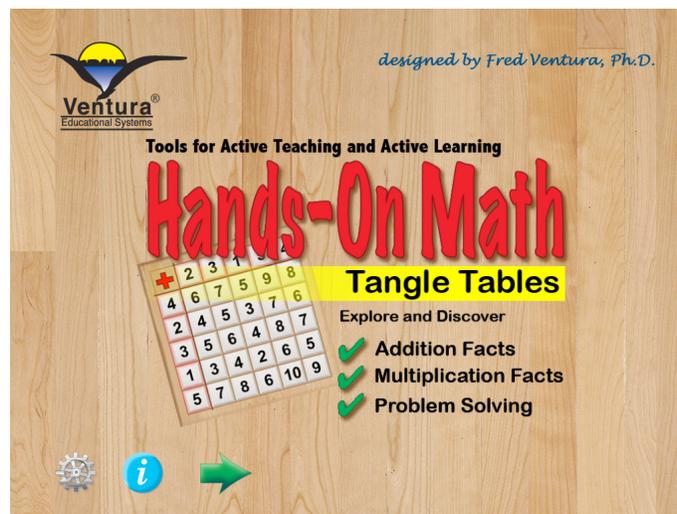
## Getting Started

Hands-On Math Tangle Tables helps student develop an understanding of basic arithmetic, especially addition and multiplication facts. The program is designed in such a way that the physical operation of the app does not interfere with the learning activity. Icons are used to provide the user with complete control over the interaction with the software features.



A Tangle Table is like a traditional addition or multiplication table but the numbers are not necessarily in order. Even so, using mathematical thinking skills the position of every number can be figured out.

Tap the Hands-On Math Tangle Table icon to launch the app.



The opening view presents the title page with three options:



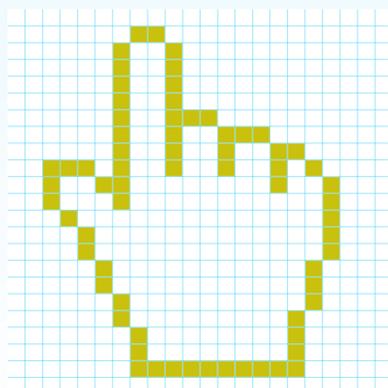
Settings - Tap this icon to control the sound, speech and view options of the app.



Info - Tap this icon to access the User's Guide where an overview of the app is presented.



Begin - Tap the green arrow to start using the Hands-On Math Tangle Tables Playground.



# Settings

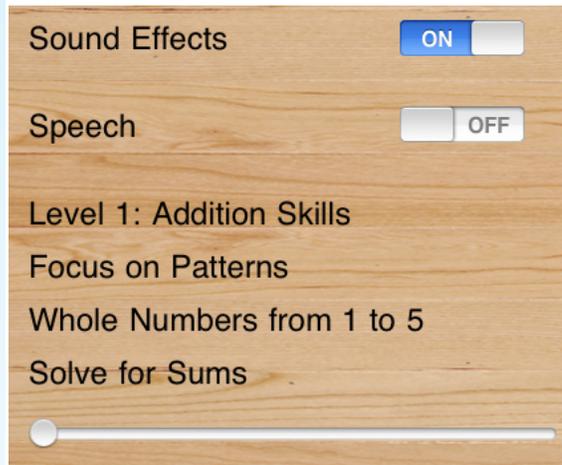


The Settings option provides control of some of the basic features of the app. Options include control for sound effects and speech.



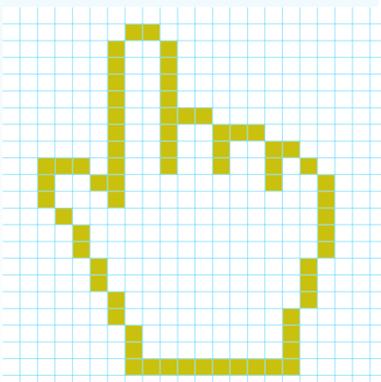
Mild sound effects are used through out the app and add a level of interest for students. When using the app with very young students teachers may wish to have the Speech option on. When Speech is 'on' the value of each Number Tile is spoken when the tile is tapped.

*Use Speech effects with very young children to reinforce the names of the numbers.*



Use the on/off switch to activate or deactivate the sound effects and speech.

The app offers 12 levels and covers addition and multiplication facts for numbers from 1 to 10 and also factoring.



## Levels 1-12

### Level 1: **Addition Skills**

Focus on Patterns  
Whole Numbers from 1 to 5  
Solve for Sums

### Level 2: **Addition Skills**

Focus on Recall of Addition Facts  
Whole Numbers from 1 to 10  
Solve for Sums

### Level 3: **Addition Skills**

Focus on Problem Solving  
Whole Numbers from 1 to 5  
Solve for Addends and Sums

### Level 4: **Addition Skills**

Focus on Problem Solving  
Whole Numbers from 1 to 10  
Solve for Addends and Sums

### Level 5: **Addition Skills**

Focus on Patterns  
Whole Numbers from 6 to 10  
Solve for Sums

### Level 6: **Addition Skills**

Focus on Problem Solving  
Whole Numbers from 6 to 10  
Solve for Addends and Sums

### Level 7: **Multiplication Skills**

Focus on Patterns  
Whole Numbers from 1 to 5  
Solve for Products

### Level 8: **Multiplication Skills**

Focus on Multiplication Facts  
Whole Numbers from 1 to 10  
Solve for Products

### Level 9: **Multiplication Skills**

Focus on Facts and Factoring  
Whole Numbers from 1 to 5  
Solve for Products and Factors

### Level 10: **Multiplication Skills**

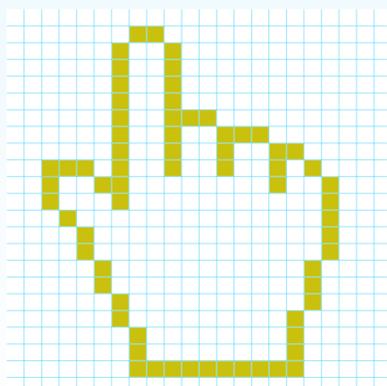
Focus on Facts and Factoring  
Whole Numbers from 1 to 10  
Solve for Products and Factors

### Level 11: **Multiplication Skills**

Focus on Facts and Factoring  
Whole Numbers from 6 to 10  
Solve for Products

### Level 12: **Multiplication Skills**

Focus on Facts and Factoring  
Whole Numbers from 6 to 10  
Solve for Products and Factors



# Tangle Table Controls and Buttons

Tap this button to show the next five available Number Tiles in the set for the selected level.



Tap the eraser to remove the last Number Tile placed on the Tangle Table



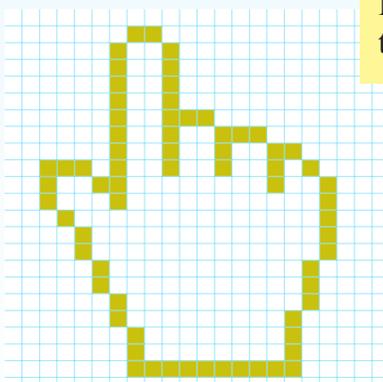
Tap the Trash Can to complete erase the Tangle Table and generate a new Tangle Table

Drag and drop Number Tiles to complete the Tangle Table.

Tap this button to show the previous five available Number Tiles in the set for the selected level.



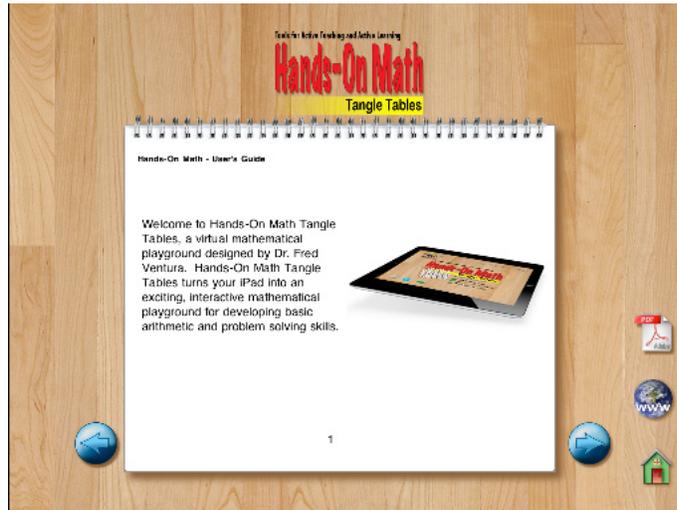
Tap Home to leave the Tangle Table Playground and return to the top level.



## In App User's Guide



Tapping the Info icon brings up the Hands-On Math User's Guide. The guide provides a quick overview to the features of the app. It serves as a quick reference to the use of the product.



Users can navigate by tapping either the right or left arrows. Swiping right or left can also be used to move to the next page or previous page.

Exit the user's guide by tapping the home icon.

*Swipe right or left to change pages or use the buttons.*



Next Page - Tap this icon to move to the next page.



Previous Page - Tap this icon to move to the next page.



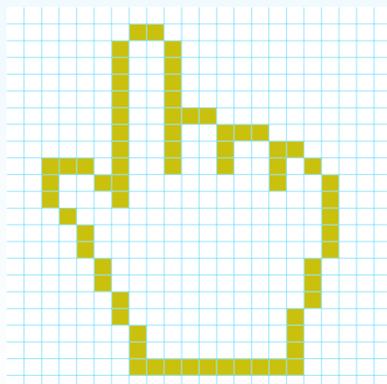
Instructor's Guide - Tap to automatically begin downloading the PDF of the Instructor's Guide from [www.venturaes.com](http://www.venturaes.com). We recommend you install the Instructor's Guide in iBooks for convenient reference.



Tap the World Wide Web icon to launch your iPad browser and view the Ventura Educational Systems website.



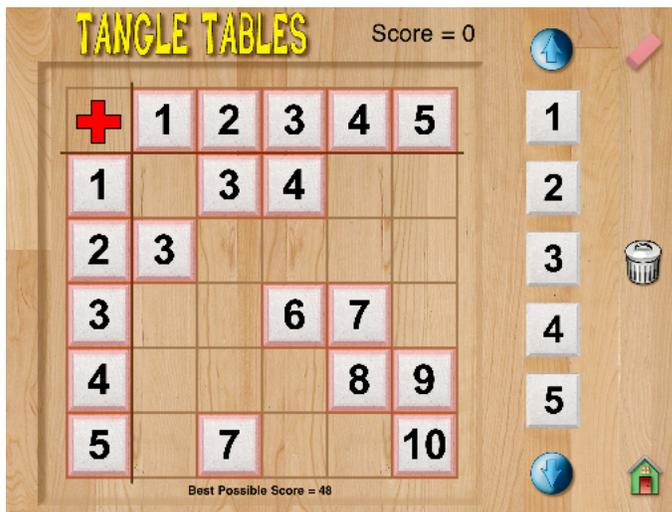
Home - Tap this icon to exit from the User's Guide.



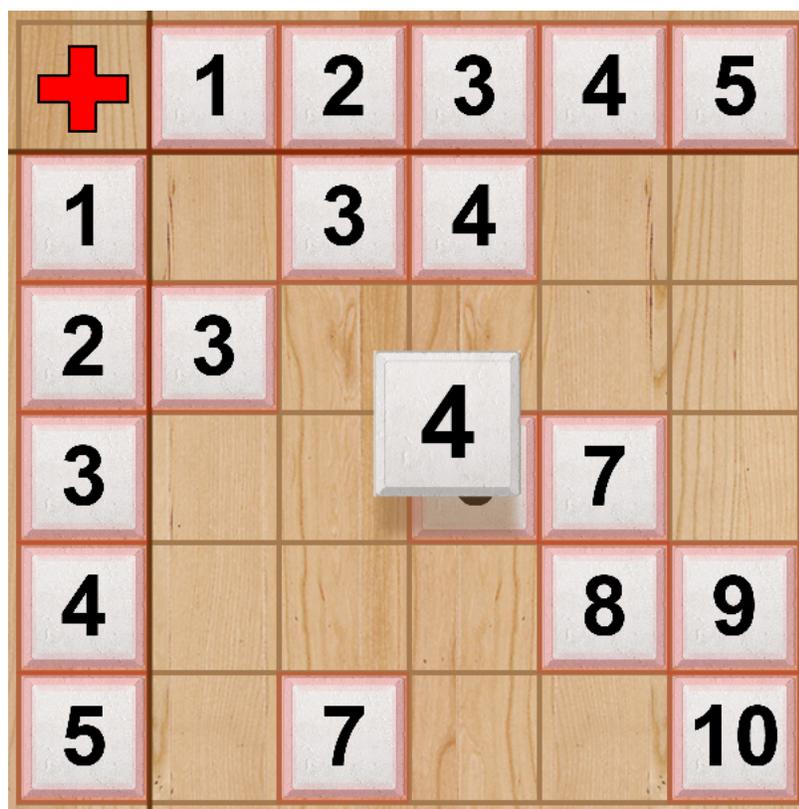
# Tangle Table Playground



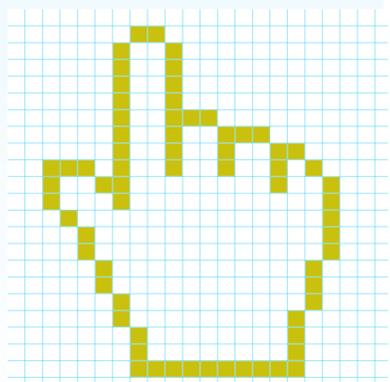
The Tangle Table Playground is where the fun begins. Tap the green arrow to get started. You will notice that in addition to the Tangle Table there are two blue arrow buttons, a set of five Number Tiles, an eraser, a trash can and a home icon.



Tile Placing Gesture: Tap one of the Number Tiles and drag and drop in one of the positions on the Tangle Table.



Notice that when a Number Tile is being moved it appears to move forward on the screen and it casts a shadow. When the tile is released by removing your finger from the screen, the tile falls into the closest spot on the Tangle Table. If a Number Tile is dropped somewhere off the Tangle Table it disappears.



## Solving a Tangle Table

*The best possible score for a Tangle Table is calculated when the puzzle is generated.*

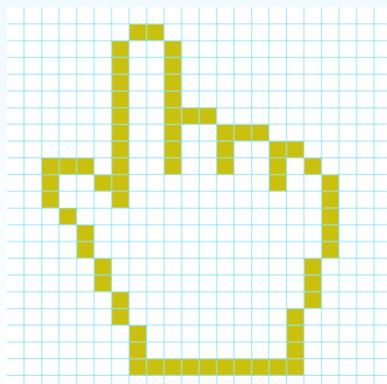
*Every time a tile is placed correctly it earns 3 points. Incorrectly placed tiles result in the loss of a single point.*

Number Tiles are dragged and dropped onto the Tangle Table until all the open positions are filled. You will notice that some tiles are tinted red. These tiles are locked and cannot be moved.

Congratulations, you completed this activity. Try another Tangle Table!



Tiles that are placed incorrectly are indicated with a question mark.



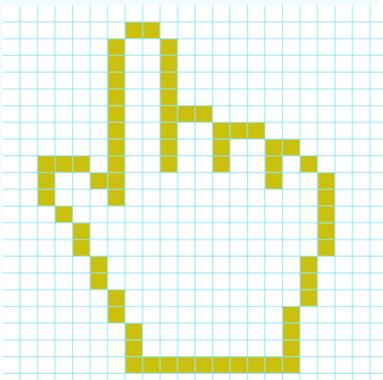
Tap the eraser icon to remove the last tile placed.



Tap the trash can icon to clear the Tangle Table and generate a new table.

## Activities

Level 1: Addition Skills 1-5 .....	15
Level 2: Addition Skills 1-10 .....	16
Level 3: Addition Skills 1-5 with Missing Addends .....	17
Level 4: Addition Skills 1-10 with Missing Addends .....	18
Level 5: Addition Skills 6-10 .....	19
Level 6: Addition Skills 6-10 with Missing Addends .....	20
Level 7: Multiplication Skills 1-5 .....	21
Level 8: Multiplication Skills 1-10 .....	22
Level 9: Multiplication Skills 1-5 with Missing Factors .....	23
Level 10: Multiplication Skills 1-10 with Missing Factors .....	24
Level 11: Multiplication Skills 6-10 .....	25
Level 12: Multiplication Skills 6-10 with Missing Factors .....	26



# Addition Skills

## 1-5



### Settings

Tap the Settings icon and use the slider to choose Level 1: Addition Facts.



Sound Effects

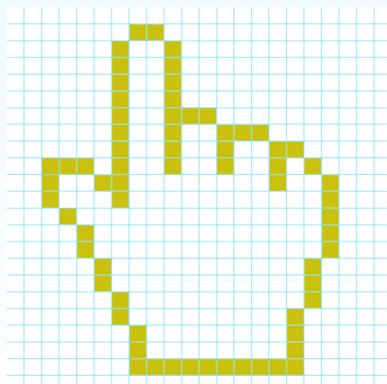


Speech

Tap the eraser to remove the last tile placed or pick up any unlocked tile and drop it off the Tangle Table to remove it.



Eraser



Let's begin exploring Tangle Tables with Level 1 Addition. After choosing Level 1 in Settings, tap the green arrow from the top level to generate a Tangle Table. Because Tangle Tables are randomly generated yours will probably look different from the one shown below. Drag and drop Number Tiles to solve the Tangle Table on your iPad.

<b>+</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>1</b>	<b>2</b>				<b>6</b>
<b>2</b>	<b>3</b>			<b>6</b>	
<b>3</b>		<b>5</b>			
<b>4</b>			<b>7</b>	<b>8</b>	
<b>5</b>			<b>8</b>		<b>10</b>

Best Possible Score = 48

Find the sums for this Tangle Table.

$1 + 2 = \square$

$3 + 4 = \square$

$1 + 3 = \square$

$3 + 5 = \square$

$1 + 4 = \square$

$4 + 1 = \square$

$2 + 2 = \square$

$4 + 2 = \square$

$2 + 3 = \square$

$4 + 5 = \square$

$2 + 5 = \square$

$5 + 1 = \square$

$3 + 1 = \square$

$5 + 2 = \square$

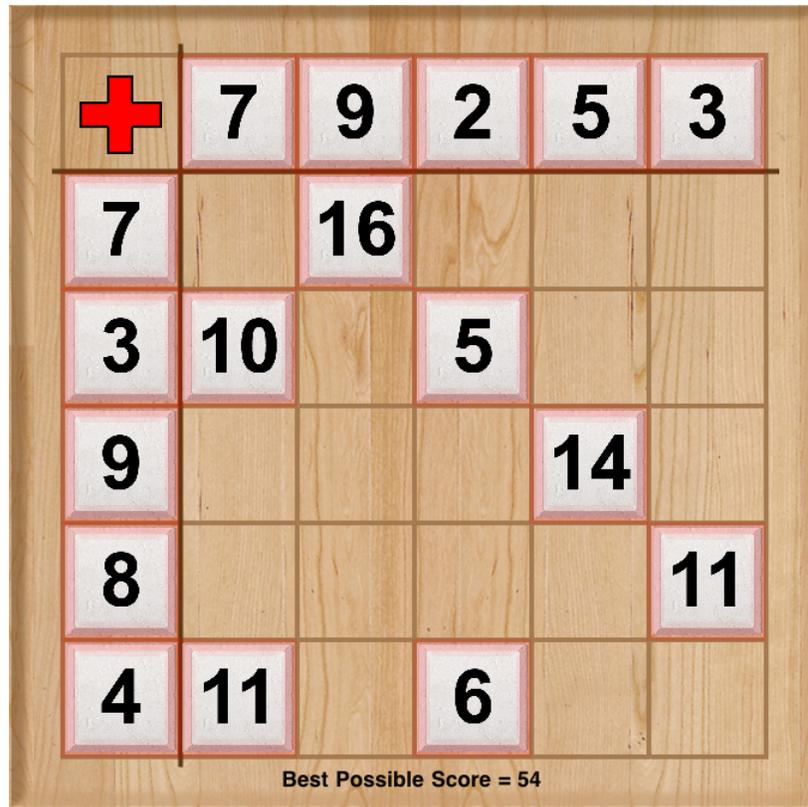
$3 + 1 = \square$

$5 + 4 = \square$

# Addition Skills

## 1-10

Let's continue exploring Tangle Tables with Level 2 Addition. After choosing Level 2 in Settings, tap the green arrow from the top level to generate a Tangle Table. Because Tangle Tables are randomly generated yours will probably look different from the one shown below. Drag and drop Number Tiles to solve the Tangle Table on your iPad.



Find the sums for this Tangle Table.

$7 + 7 = \square$

$7 + 2 = \square$

$7 + 5 = \square$

$7 + 3 = \square$

$3 + 9 = \square$

$3 + 3 = \square$

$9 + 7 = \square$

$9 + 9 = \square$

$9 + 2 = \square$

$9 + 3 = \square$

$8 + 7 = \square$

$8 + 9 = \square$

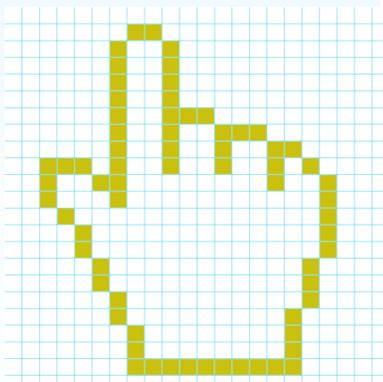
$8 + 2 = \square$

$8 + 5 = \square$

$4 + 9 = \square$

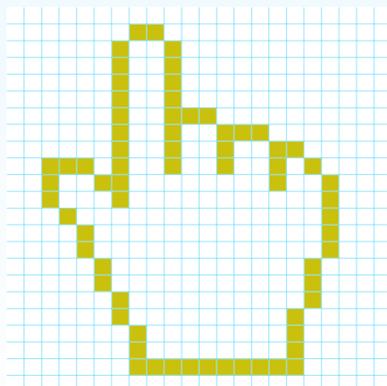
$4 + 5 = \square$

$4 + 3 = \square$



## Addition Skills 1-5 with Missing Addends

The first row contains the numbers 1-5, but not necessarily in that order. The first column also contains the numbers 1-5, but not necessarily in that order. Each of the numbers 1-5 can only be used once. The challenge is to fill in all the addends and sums.



Level 3 Addition involves missing addends. The addends at the beginning of each row and the top of each column are missing. By studying the Tangle Table you can figure out what those numbers should be. Try it!

Best Possible Score = 75

What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Sum
1	6	7
2	5	7
3	4	7
4	3	7
5	2	7
6	1	7

Use this thinking strategy to solve Level 3 Tangle Tables. Move the Number Tiles around while trying solutions.

Remember that in the left column and top row Number Tiles are used only once.

# Addition Skills 1-10 with Missing Addends

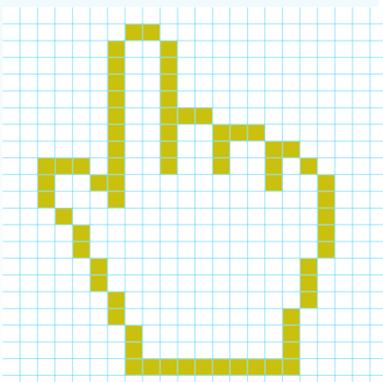
Level 4 Addition involves missing addends with numbers from 1 through 10. The addends at the beginning of each row and the top of each column are missing. By studying the Tangle Table you can figure out what those numbers should be. Try it!

Best Possible Score = 75

What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Sum
1	13	14
2	12	14
3	11	14
4	10	14
5	9	14
6	8	14
7	7	14

$a$	$b$	Sum
8	6	14
9	5	14
10	4	14
11	3	14
12	2	14
13	1	14



Use this thinking strategy to solve Level 5 Tangle Tables. Move the Number Tiles around while trying solutions.

Remember that in the left column and top row Number Tiles are used only once.

# Addition Skills

## 6-10

At Level 5 Addition Tangle Tables are generated using numbers from 6 through 10. The addends at the beginning of each row and the top of each column are given. Complete the Tangle Table by filling in the missing numbers.

<b>+</b>	<b>7</b>	<b>9</b>	<b>10</b>	<b>8</b>	<b>6</b>
<b>9</b>	<b>16</b>		<b>19</b>		
<b>10</b>		<b>19</b>			<b>16</b>
<b>7</b>	<b>14</b>			<b>15</b>	
<b>8</b>			<b>18</b>		<b>14</b>
<b>6</b>		<b>15</b>		<b>14</b>	

Best Possible Score = 45

Find the sums for this Tangle Table.

$9 + 9 =$

$7 + 6 =$

$9 + 8 =$

$8 + 7 =$

$9 + 6 =$

$8 + 9 =$

$10 + 7 =$

$8 + 8 =$

$10 + 10 =$

$6 + 7 =$

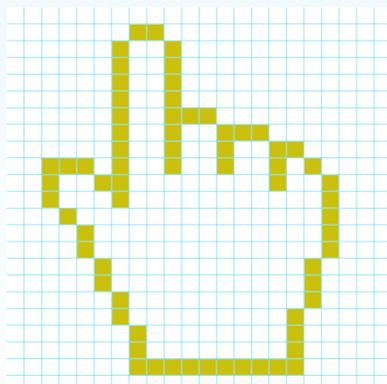
$10 + 8 =$

$6 + 10 =$

$7 + 9 =$

$6 + 6 =$

$7 + 10 =$



## Addition Skills 6-10 with Missing Addends

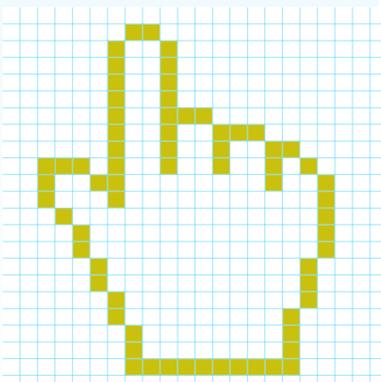
Level 6 Addition involves missing addends with numbers from 1 through 10. The addends at the beginning of each row and the top of each column are missing. By studying the Tangle Table you can figure out what those numbers should be. Try it!

Best Possible Score = 81

What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Sum
1	13	14
2	12	14
3	11	14
4	10	14
5	9	14
6	8	14
7	7	14

$a$	$b$	Sum
8	6	14
9	5	14
10	4	14
11	3	14
12	2	14
13	1	14



Use this thinking strategy to solve Level 6 Tangle Tables. Move the Number Tiles around while trying solutions.

Remember that in the left column and top row Number Tiles are used only once.

# Multiplication Skills

## 1-5

Level 7 Multiplication involves finding products for numbers from 1 through 5. The factors at the beginning of each row and the top of each column are given. Drag and drop Number Tiles to solve the Tangle Table on your iPad.

<b>×</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>1</b>		<b>2</b>			<b>5</b>
<b>2</b>	<b>2</b>			<b>8</b>	
<b>3</b>	<b>3</b>				<b>15</b>
<b>4</b>		<b>8</b>	<b>12</b>		
<b>5</b>			<b>15</b>	<b>20</b>	

Best Possible Score = 45

Find the products for this Tangle Table.

$1 \times 1 =$

$3 \times 4 =$

$1 \times 3 =$

$4 \times 1 =$

$1 \times 4 =$

$4 \times 4 =$

$2 \times 2 =$

$4 \times 5 =$

$2 \times 3 =$

$5 \times 1 =$

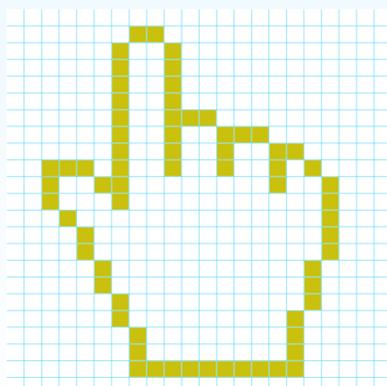
$2 \times 5 =$

$5 \times 2 =$

$3 \times 2 =$

$5 \times 5 =$

$3 \times 3 =$



# Multiplication Skills

## 1-10

Level 8 Multiplication involves finding products for numbers from 1 through 10. The factors at the beginning of each row and the top of each column are given. Drag and drop Number Tiles to solve a Level 8 Tangle Table on your iPad. Here is a randomly generated sample.

Best Possible Score = 45

Find the products for this Tangle Table.

$7 \times 10 = \square$

$2 \times 2 = \square$

$7 \times 3 = \square$

$8 \times 7 = \square$

$7 \times 8 = \square$

$8 \times 2 = \square$

$4 \times 7 = \square$

$8 \times 8 = \square$

$4 \times 3 = \square$

$6 \times 10 = \square$

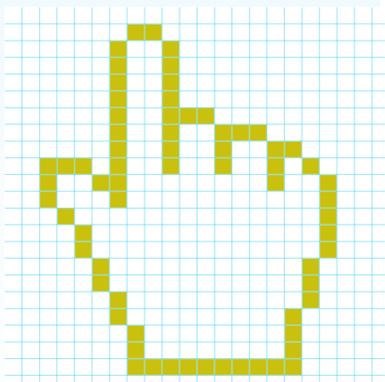
$4 \times 2 = \square$

$6 \times 3 = \square$

$2 \times 10 = \square$

$6 \times 8 = \square$

$2 \times 7 = \square$



# Multiplication Skills

## 1-5 with Missing Factors

The first row contains the numbers 1-5, but not necessarily in that order. The first column also contains the numbers 1-5, but not necessarily in that order. Each of the numbers 1-5 can only be used once. The challenge is to fill in all the factors and products.

Level 9 Multiplication involves finding products for numbers from 1 through 5. The factors at the beginning of each row and the top of each column are missing. Drag and drop Number Tiles to solve a Level 9 Tangle Table on your iPad. Here is a randomly generated sample.

Best Possible Score = 75

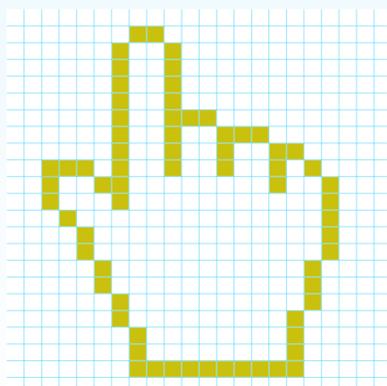
What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Product
2	4	8
4	2	8

Because the range of numbers is from 1 to 5, 2 and 4 are the only possible factors for 8. Look down the column to find 6.

$a$	$b$	Product
2	3	6
3	2	6

Since 2 is a factor of both 8 and 6,  $b$  must be equal to 2. Use this type of mathematical thinking to solve Level 9 Tangle Tables.



# Multiplication Skills

## 1-10 with Missing Factors

The first row contains the numbers 1-10, but not necessarily in that order. The first column also contains the numbers 1-10, but not necessarily in that order. Each of the numbers 1-10 can only be used once. The challenge is to fill in all the factors and products.

Level 10 Multiplication involves finding products for numbers from 1 through 10. The factors at the beginning of each row and the top of each column are missing. Drag and drop Number Tiles to solve a Level 10 Tangle Table on your iPad. Here is a randomly generated sample.

Best Possible Score = 78

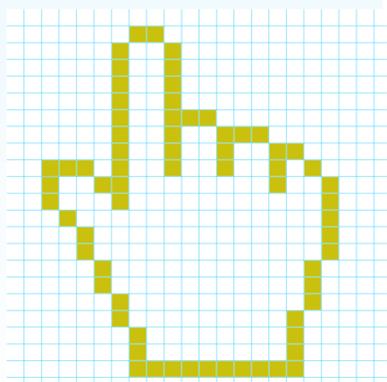
What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Product
3	9	8
9	3	8

Because the range of numbers is from 1 to 10, 3 and 9 are the only possible factors for 27. Look down the column to find 81.

$a$	$b$	Product
9	9	81

Since 9 is a factor of both 27 and 81,  $b$  must be equal to 9. Use this type of mathematical thinking to solve Level 10 Tangle Tables.



# Multiplication Skills

## 6-10

Level 11 Multiplication involves finding products for numbers from 6 through 10. The factors at the beginning of each row and the top of each column are given. Drag and drop Number Tiles to solve a Level 11 Tangle Table on your iPad. Here is a randomly generated sample.

<b>×</b>	<b>7</b>	<b>9</b>	<b>6</b>	<b>10</b>	<b>8</b>
<b>8</b>			<b>48</b>		
<b>10</b>	<b>70</b>			<b>100</b>	
<b>6</b>		<b>54</b>			<b>48</b>
<b>7</b>		<b>63</b>			<b>56</b>
<b>9</b>	<b>63</b>			<b>90</b>	

Best Possible Score = 48

Find the products for this Tangle Table.

$8 \times 7 =$

$6 \times 6 =$

$8 \times 9 =$

$6 \times 10 =$

$8 \times 10 =$

$7 \times 7 =$

$8 \times 8 =$

$7 \times 6 =$

$10 \times 9 =$

$7 \times 10 =$

$10 \times 6 =$

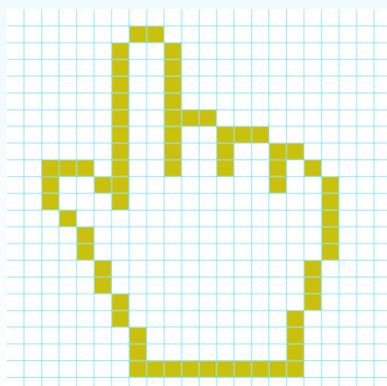
$9 \times 9 =$

$10 \times 8 =$

$9 \times 6 =$

$6 \times 7 =$

$9 \times 8 =$



# Multiplication Skills

## 6-10 with Missing Factors

Level 12 Multiplication involves finding products for numbers from 6 through 10. The factors at the beginning of each row and the top of each column are missing. Drag and drop Number Tiles to solve a Level 12 Tangle Table on your iPad. Here is a randomly generated sample.

Best Possible Score = 78

What are the possible numbers for  $a$  and  $b$ ?

$a$	$b$	Product
7	10	70
10	7	70

Because the range of numbers is from 6 to 10, 7 and 10 are the only possible factors for 70. Look down the column to find 90.

$b$	$c$	Product
9	10	90
10	9	90

Since 10 is a factor of both 70 and 90,  $b$  must be equal to 10. Use this type of mathematical thinking to solve Level 12 Tangle Tables.

