



**Tools for Active Teaching and Active Learning**

# Hands-On Math

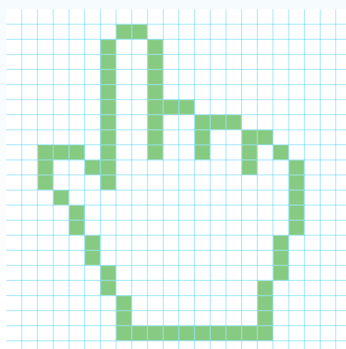
## Line Design



**Explore and Discover**

- ✓ **Basic Geometry**
- ✓ **Angles**
- ✓ **Aestheometry**

## Instructor's Guide



**Ventura Educational Systems**

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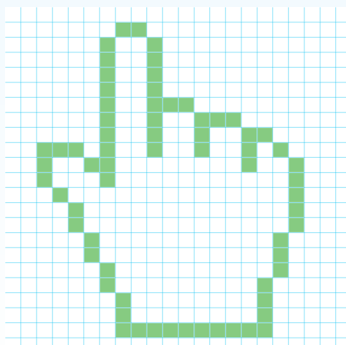
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|---|--|
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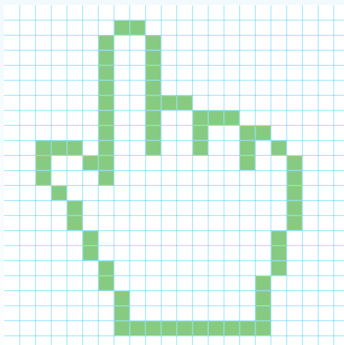
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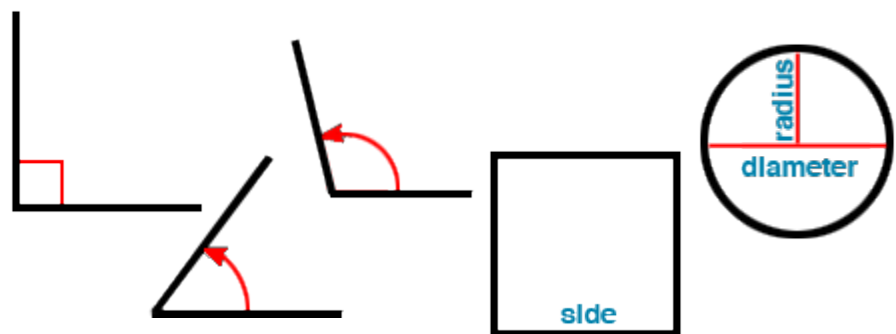


## Overview

The use of a Hands-On Math Line Design is a great way to enhance any math curriculum and to give students an insight into basic geometry. The formal study of geometry is often attributed to the Greeks where simple drawing tools, a straight line, and an arc were used to explore the properties of a variety of figures. Descartes furthered the study of geometry with his development of Cartesian or analytic geometry. Descartes used algebra to describe geometry. He invented the convention of representing positions using x, y, and z. In addition to Descartes rectangular coordinate system, points can be located using polar coordinates. In the polar coordinate system a point is defined by the distance from the origin and the angle of the arc. Hands-On Math Line Design is designed to help children develop an understanding of these concepts and more. In order for a child to develop a meaningful understanding of mathematics it is essential that the child know the underlying concepts of basic geometry. After a student has developed clear understanding of these ideas problem solving related to geometry can be introduced.

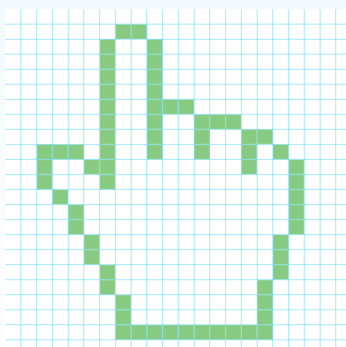
Learning basic geometry involves the study of lines, line segments, shapes and symmetry. The introductory study of geometry includes learning about the properties of shapes and basic concepts such as congruence, similarity, symmetry as well as area and perimeter. Children learn that shapes have properties that can be described with geometric terminology.

The Hands-On Math Line Design app turns your iPad into a interactive tabletop surface that we call a Playground. The Playground provides the child with an opportunity to freely explore the basic concepts of geometry. The Playground provides a variety of patterns to serve as starting point for creative mathematical discovery.



Research has shown that children learn best through active involvement in the learning process. Hands-On Math Line Design is designed to be a tool that teachers can use for active teaching and active learning. By design it encourages creative mathematical thinking and artist expression. Math manipulative devices can be a rich source of teaching strategies for problem solving and can be very helpful in developing an intuitive understanding of mathematical concepts. The Hands-On Math series

*The relationship between geometry and art has a long history, and exploring the role that geometry plays in creating art is a fascinating topic for students to creatively explore.*

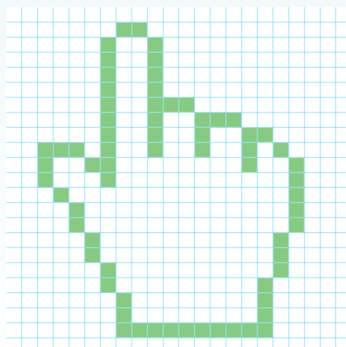


suggests ways in which concrete learning experiences can be extended to a representational level and still remain manipulative and interactive.

This guide consists of two sections. The first part is written for the teacher and explains the functions of the app and options available. It presents ideas for instructional strategies that can be implemented with each simulated manipulative device. The second section of the manual is a set of curriculum-based activities that are designed to help the teacher in using the Hands-On Math app. These activities have been developed for elementary and middle school age children and are arranged by order of grade level where the concepts are typically introduced. Teachers will want to decide what is the best sequence for using the materials with their particular group of students. Each lesson is aimed at specific mathematical objectives including counting, representing numbers using the place value system, addition and subtraction with regrouping. Each activity is meant to be a beginning. Teachers will want to encourage the children to explore extensions of each activity with different examples. Orally discussing each activity will help to foster higher level thinking.

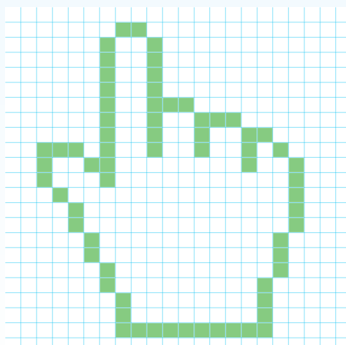
Hands-On Math Line Design is a starting point. Learning should be fun and as students work with the app, it is my intention that they will begin discussing, sharing and creatively exploring mathematics.

-- Fred Ventura, Ph.D.



## Introduction to Hands-On Math Line Design

*Piaget's theory of cognitive development is a comprehensive theory about the nature and development of human intelligence.*



Approaches to the teaching of mathematics that rely heavily on one methodology are inherently weak and unlikely to produce optimal results. Educators have found that teaching strategies must adapt to accommodate new discoveries which expand our understanding of the learning process and new technologies which expand our delivery systems.

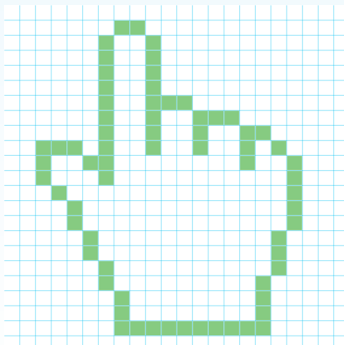
According to learning theory, children learn best when they are actively involved in the learning process. There are many ways to do this but one example is having children work in small groups in a laboratory/discovery situation. Small group instruction encourages variation in teaching methodology. Variation in the way in which material is presented serves the instructional process since one particular methodology may not be best for all children. Different children respond differently to a particular educational approach. The same methodology that is appropriate for one content area or developmental stage may not be appropriate in a different content area or with children who are at a different developmental stage.

For learning mathematics an active teaching and active learning situation is a very desirable educational environment. To create it the teacher must be aware of the behavioral characteristics of the students with regard to mathematics, must be knowledgeable in the particular skills which are being taught and must be able to draw upon diverse strategies in order to decide which is the most appropriate for fostering the development of the targeted mathematical concepts.

In general, educational psychologists believe that the ability of children to learn passes through developmental stages. Each stage is characterized by particular behaviors. In the early stages learning is tied to perceptual responses. As the child matures, abstract reasoning becomes possible and concrete models are useful for laying the conceptual groundwork for new ideas, but once a concept has been internalized the concrete models are no longer necessary. The work of Swiss psychologist, Jean Piaget, has contributed a great deal to support this theory, and to foster the development of educational strategies which are consistent with the theory.

## Using an iPad in a Manipulative Approach to Math

*We use the term “playground” to convey the open-ended, discovery approach to learning that this tool was designed to support.*



Hands-On Math Line Design combines and extends the use of concrete materials for teaching mathematics to the touch-based interactive environment of the Apple® iPad™. When used in conjunction with actual manipulative devices the app offers a unique set of strategies for active learning. While using the app students can draw upon concepts developed from concrete experiences that were gained using manipulative devices and will work with the same concepts in a more representational manner using the app. In this way the child's concrete mathematical knowledge is used to help make a transition to a representational stage and serves as a foundation for the development of abstract mathematical thinking skills.

Once mathematical concepts have been internalized by the child in a concrete way, the stage is set for a deeper understanding of the more formal, abstract axioms of higher mathematics.

Hands-On Math Line Design provides geometric patterns to explore designs created by iteration. Using only a straight line segment a great deal of geometry can be learned. The concepts studied in this curriculum are the foundations of geometry. The app simulates a popular activity where a needle and embroidery thread is used on cardboard to create designs, however the opportunities for learning are augmented by the artificially intelligent feedback provided by the software. The instructional approaches that result from creating an open-ended area for free exploration and discovery are exciting. On the Line Design Playground students manipulate a supply of colored line segments to create elaborate geometric designs and in the process learn about the properties of shapes, coordinate systems for locating points and other principles of geometry.

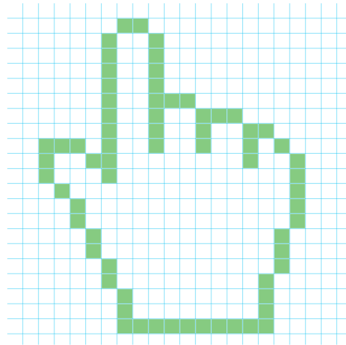
Using the Line Design Playground students can use gestures to draw line segments. The iPad app will automatically replicate the design. The author and designer coined the term, “artificially intelligent math manipulative” to describe how using Hands-On Math Line Design differs from the concrete manipulative devices traditionally used in classrooms. The Hands-On Math Line Design Playground provides intelligent mathematical feedback as the student manipulates the device.

The Hands-On Math Line Design Playground can also be effectively used with lessons that present mathematical concepts in a structured way. Initially teachers may want to provide ample free exploration time and then after the students have become familiar with the product, direct students into more structured investigations. Students will make discoveries and when they do teachers should encourage them to share their discoveries with others in their group.

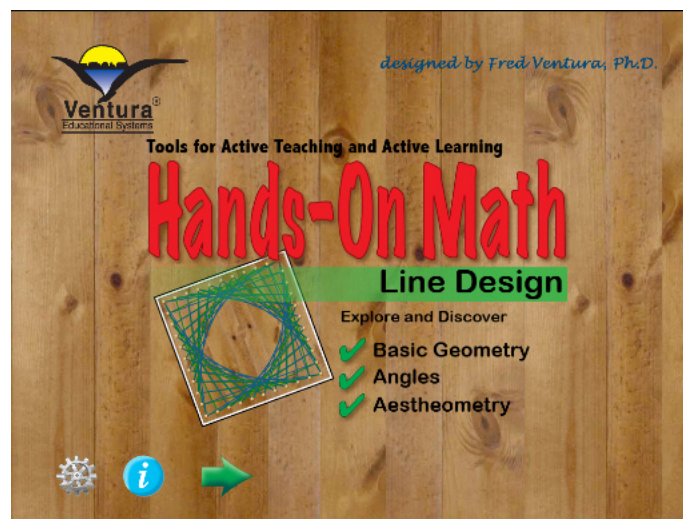


## Getting Started

Hands-On Math Line Design helps student develop an understanding of basic geometry. The program is designed in such a way that the physical operation of the app does not interfere with the learning activity. Icons are used to provide the user with complete control over the interaction with the software features.



Tap the Hands-On Math Line Design icon to launch the app.



The opening view presents the title page with three options:



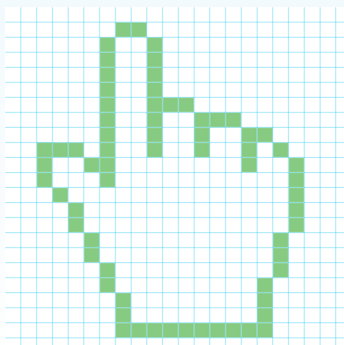
Settings - Tap this icon to control the sound, speech and view options of the app.



Info - Tap this icon to access the User's Guide where an overview of the app is presented.



Begin - Tap the green arrow to start using the Hands-On Math Line Design Playground.





## Settings

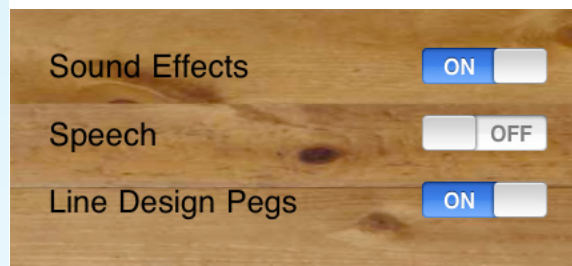


The Settings option provides control of some of the basic features of the app. Options include control for sound effects and speech.



Mild sound effects are used through out the app and add a level of interest for students. When using the app with very young students teachers may wish to have the Speech option on. When Speech is on the name of each color is pronounced when a color splash is tapped.

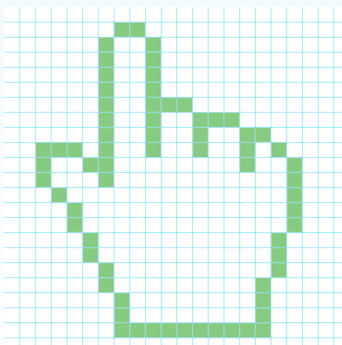
*Use Speech effects with very young children to help them learn the names of colors.*



Use the on/off switches to activate or deactivate the sound effects, speech or the display of pegs.

The pegs used to mark the beginning and ending points for line segments can be turned on or off. When the Line Design pegs are hidden a simple grid showing either polar or rectangular coordinates is displayed.

When speech is 'On' color names are pronounced as they are selected.



## Drawing Tools

Tap a color splash to select a color.



*Black*



*Orange*



*Blue*



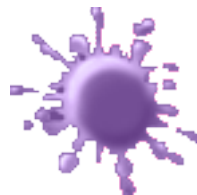
*Red*



*Green*



*White*



*Purple*



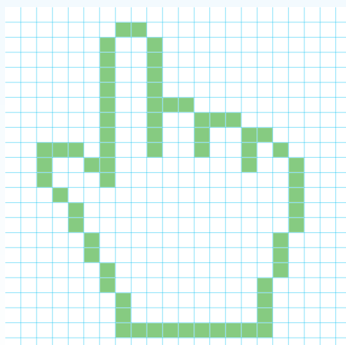
*Yellow*

Tap a line segment icon to draw line segments from one point to another.  
Tap the labelling icon and tap to label the points with the letters A-Z in sequence.

**Line Segment Icon**



**Labelling Icon**



## In App User's Guide



Tapping the info icon brings up the Hands-On Math User's Guide. The guide provides a quick overview to the features of the app. It serves as a quick reference to the use of the product.



Users can navigate by tapping either the right or left arrows. Swiping right or left can also be used to move to the next page or previous page.

Exit the user's guide by tapping the home icon.

*Swipe right or left to change pages or use the buttons.*



Next Page - Tap this icon to move to the next page.



Previous Page - Tap this icon to move to the next page.



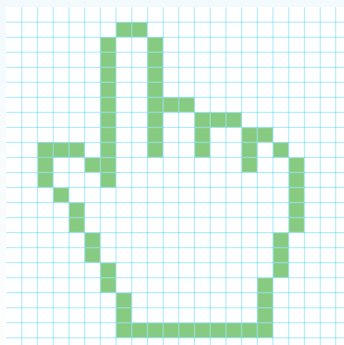
Instructor's Guide - Tap to automatically begin downloading the PDF of the Instructor's Guide from [www.venturaes.com](http://www.venturaes.com). We recommend you install the Instructor's Guide in iBooks for convenient reference.



Tap the World Wide Web icon to launch your iPad browser and view the Ventura Educational Systems website.



Home - Tap this icon to exit from the User's Guide.



## Line Design Playground



The Line Design Playground is where the fun begins. Tap the green arrow to get started. You will notice that in addition to the design area there are eight color splashes, an eraser and a trash can, a grid icon, a labelling icon, a line segment icon, an xy-axes icon and note card.

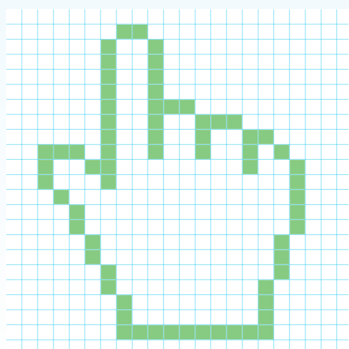


Drawing Gesture: Tap black and then tap the line segment tool. Touch the top left peg on the Line Design pattern and slide your figure to the first bottom peg as shown here:

Line Segment



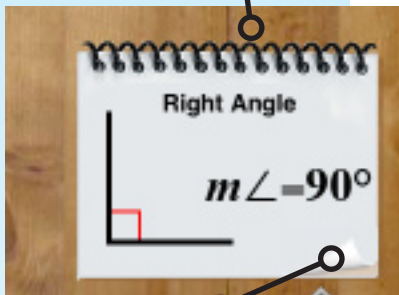
When you release your finger a diagonal line is drawn and the Go! icon appears in the tool area. Tap Go! to replicate the line segment. Experiment with the other colors and new line segments to create a mathematical design.





## Drawing with the Right Angle Pattern (90°)

Tap here to flip to the previous card.



Tap here to flip to the next card.

Select the right angle pattern by tapping the note card. Tap red to choose a color and tap the line segment icon. Drag to draw a line as shown here.



# GO!

Tap Go! to replicate the line and create the Line Design.

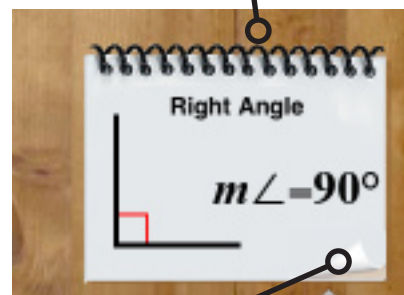


Tap the eraser icon to remove a line.

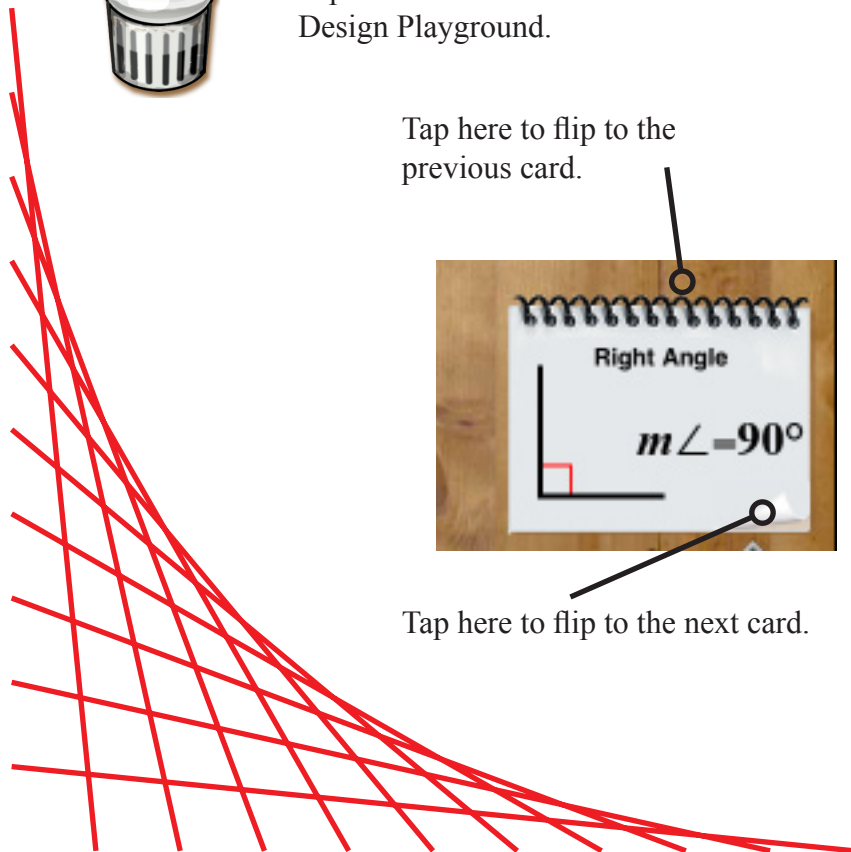
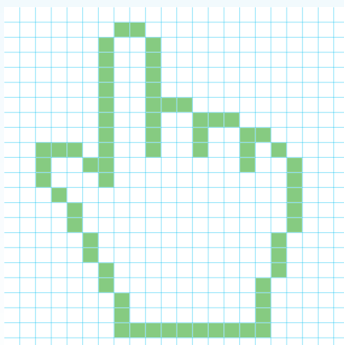


Tap the trash can icon to clear the Line Design Playground.

Tap here to flip to the previous card.

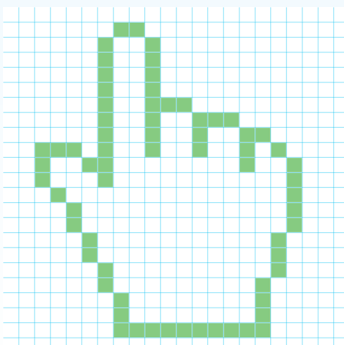


Tap here to flip to the next card.



## Activities

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## Line Design Patterns

Line Segment



Let's begin exploring the Hands-On Math Line Design Playground by flipping through the note cards to see the options.

### Right Angle



$$m\angle = 90^\circ$$

Five different patterns can be selected: Right Angle, Acute Angle, Obtuse Angle, Square or Circle.

### Acute Angle



$$m\angle < 90^\circ$$

### Obtuse Angle



$$m\angle > 90^\circ$$

For angle patterns there are 10 points along each leg. The square pattern has 10 points along each side.

### Square



$$P = 4s$$

$$A = s^2$$

### Circle

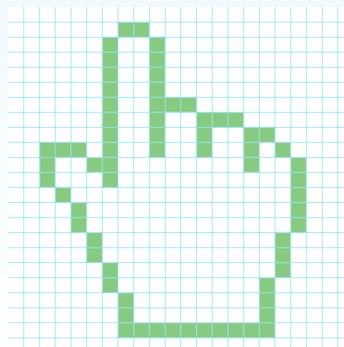


$$C = \pi a$$

$$A = \pi r^2$$

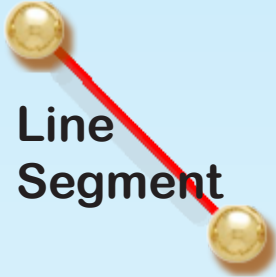
$$\pi \approx 3.14$$

When the circle pattern is selected a slider control allows the user to set the number of equidistant points along the circumference.





## Exploring Line Designs



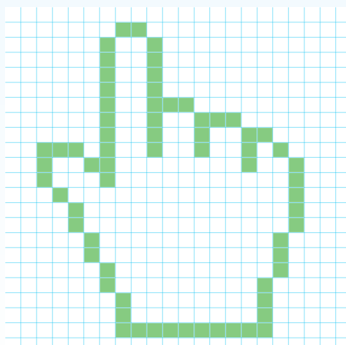
**Line  
Segment**



**Red  
Splash**

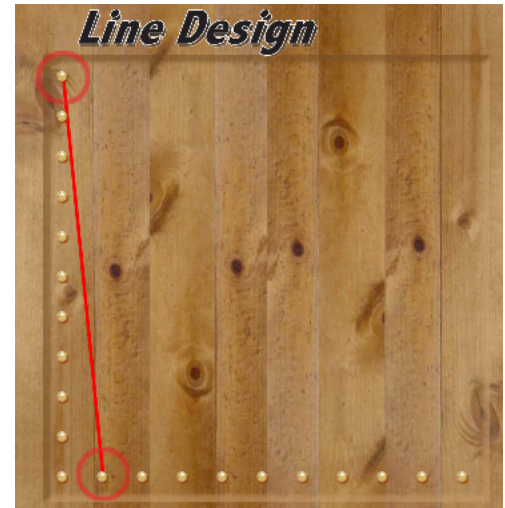


**Trash Can**

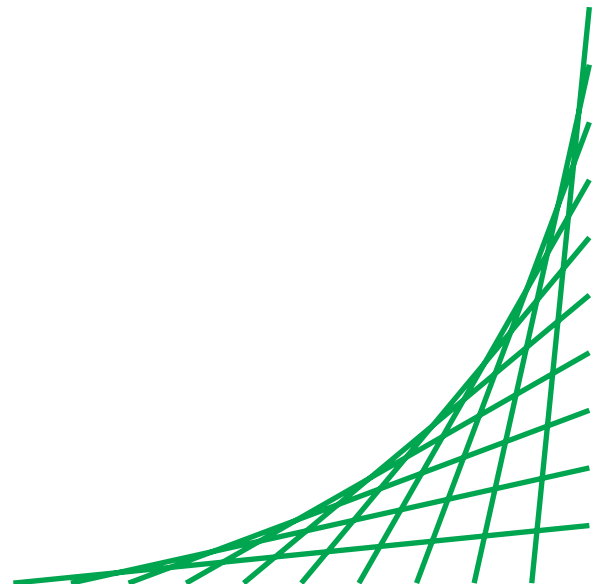
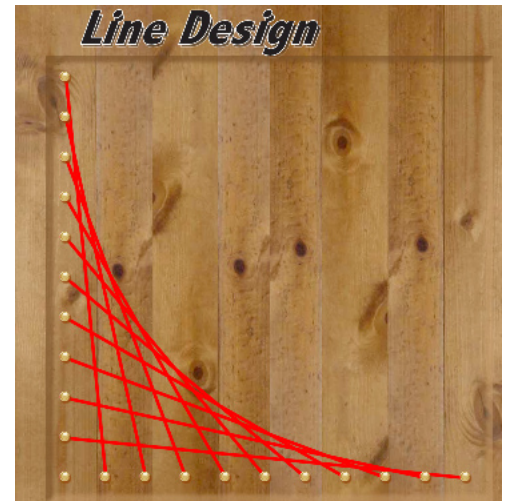


Use the Line Design Playground to complete these activities.

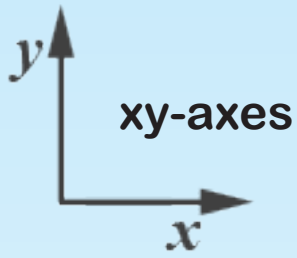
1. Draw a line segments on the right angle pattern as shown here. To begin tap the line segment tool and then tap the red color splash. Draw the line segment using by dragging from one point to another.



2. Once the line segment has been drawn, the Go! button appears in the tool area. Tap Go! to create the design.
3. Tap the trash can to completely erase the design.
4. Create other designs on the right angle pattern using different colors.



$$x + y =$$



- (0,9) - (1,0)
- (0,8) - (2,0)
- (0,7) - (3,0)
- (0,6) - (4,0)
- (0,5) - (5,0)
- (0,4) - (6,0)
- (0,3) - (7,0)
- (0,2) - (8,0)
- (0,1) - (9,0)

Tap the xy-axes icon to label the coordinates. Choose any color for this design.



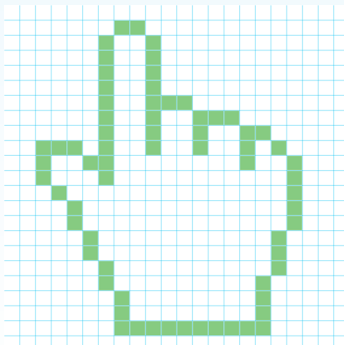
Let's do mental math and find the sum of the  $x$  and  $y$  coordinates for this design. The gray boxes represent the  $x$  and  $y$  coordinates for each line segment. What is the sum of each coordinate pair?

$$6 + 3 + 5 + 4 + 7 = \boxed{\phantom{00}}$$

10

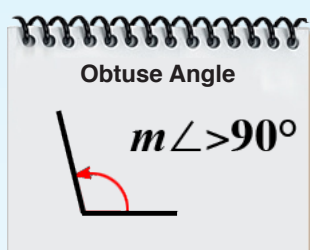
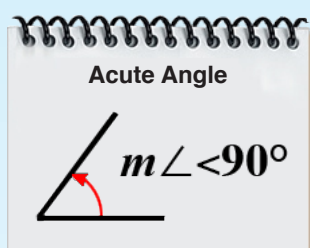
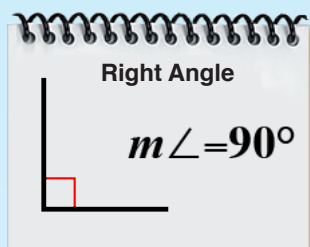
*Tip for Math Wizards!*

*When adding a set of numbers look for tens to make the computation easier.*

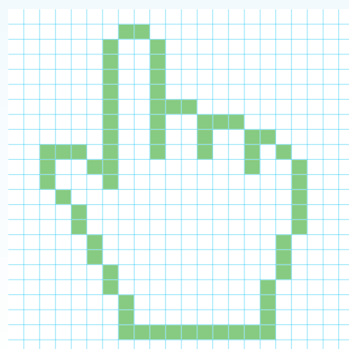


|     |   |   |   |   |   |   |   |   |   |     |
|-----|---|---|---|---|---|---|---|---|---|-----|
| $y$ | 9 |   |   |   |   |   |   |   |   |     |
| 8   |   |   |   |   |   |   |   |   |   |     |
| 7   |   |   |   |   |   |   |   |   |   |     |
| 6   |   |   |   |   |   |   |   |   |   |     |
| 5   |   |   |   |   |   |   |   |   |   |     |
| 4   |   |   |   |   |   |   |   |   |   |     |
| 3   |   |   |   |   |   |   |   |   |   |     |
| 2   |   |   |   |   |   |   |   |   |   |     |
| 1   |   |   |   |   |   |   |   |   |   |     |
|     | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | $x$ |

# Classifying Angles



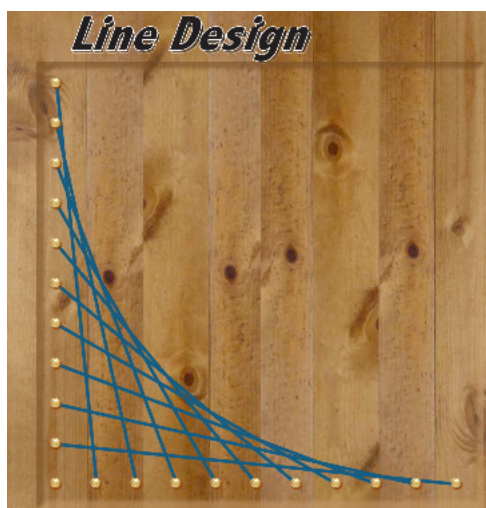
A protractor is a tool that used to measure angles.



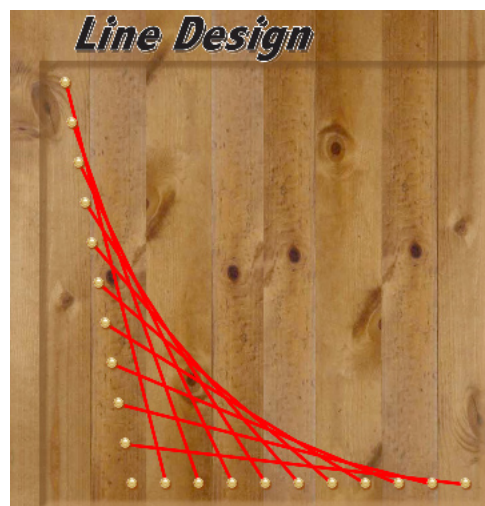
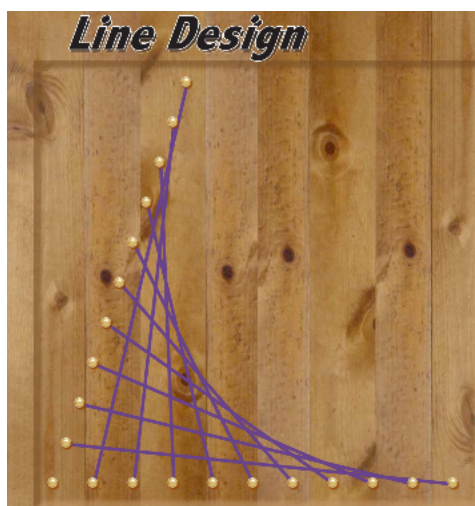
Angles can be classified as right angles, acute angles or obtuse angles.

*Angles that measure  $90^\circ$  are called right angles. Acute angles have measures that are less than  $90^\circ$  and obtuse angles measure greater than  $90^\circ$ .*

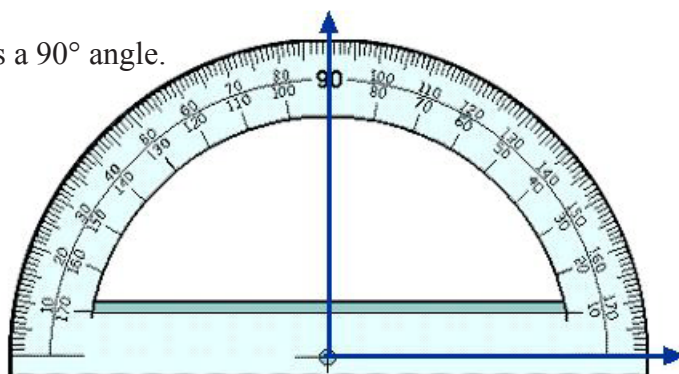
Tap the dog ear icon on the right corner of the note pad to change the design pattern.



1. Create a line design on the right angle pattern.
2. Create a similar design with a different color on the acute angle pattern.
3. Create a similar design with a different color on the obtuse angle pattern.

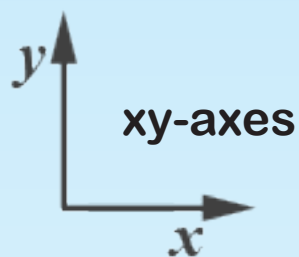


The protractor shows a  $90^\circ$  angle.

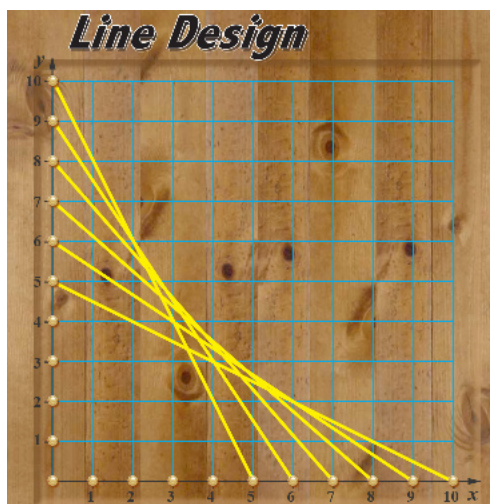




## Coordinate Pairs



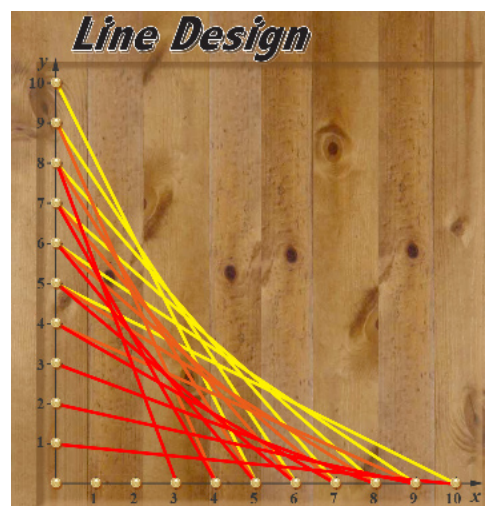
Choose the right angle pattern for this activity. Tap the xy-axes icon to label the coordinates.



1. Select yellow and draw a line segment from (0,10) to (5,0).
2. Select orange and draw a line segment from (0,9)-(4,0)
3. Select red and draw a line segment from (0,8)-(3,0).

Does your design look like this one?

This design is made up of 6 yellow line segments, 7 orange line segments and 8 red line segments. How many line segments in all are used to make this design?

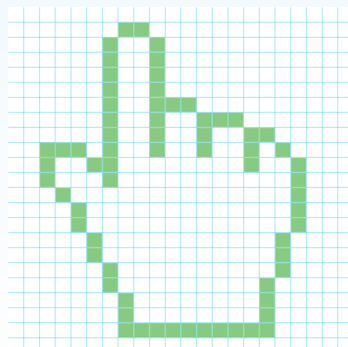


$$6 + 7 + 8 = \square$$

Choose a partner but sit facing each other so your partner can't see your iPad screen. Create a similar design but don't let your partner see your design. Write a set of instructions for your partner using specific colors and coordinate pairs.

Use mathematical terms such as *line segment* and *coordinate pair* to tell your partner how to copy your design. Can your partner recreate your design accurately?

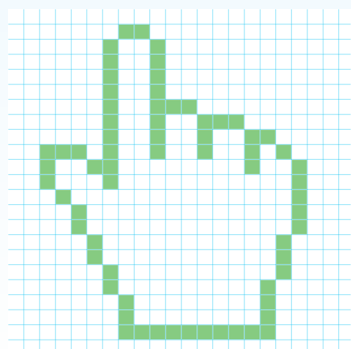
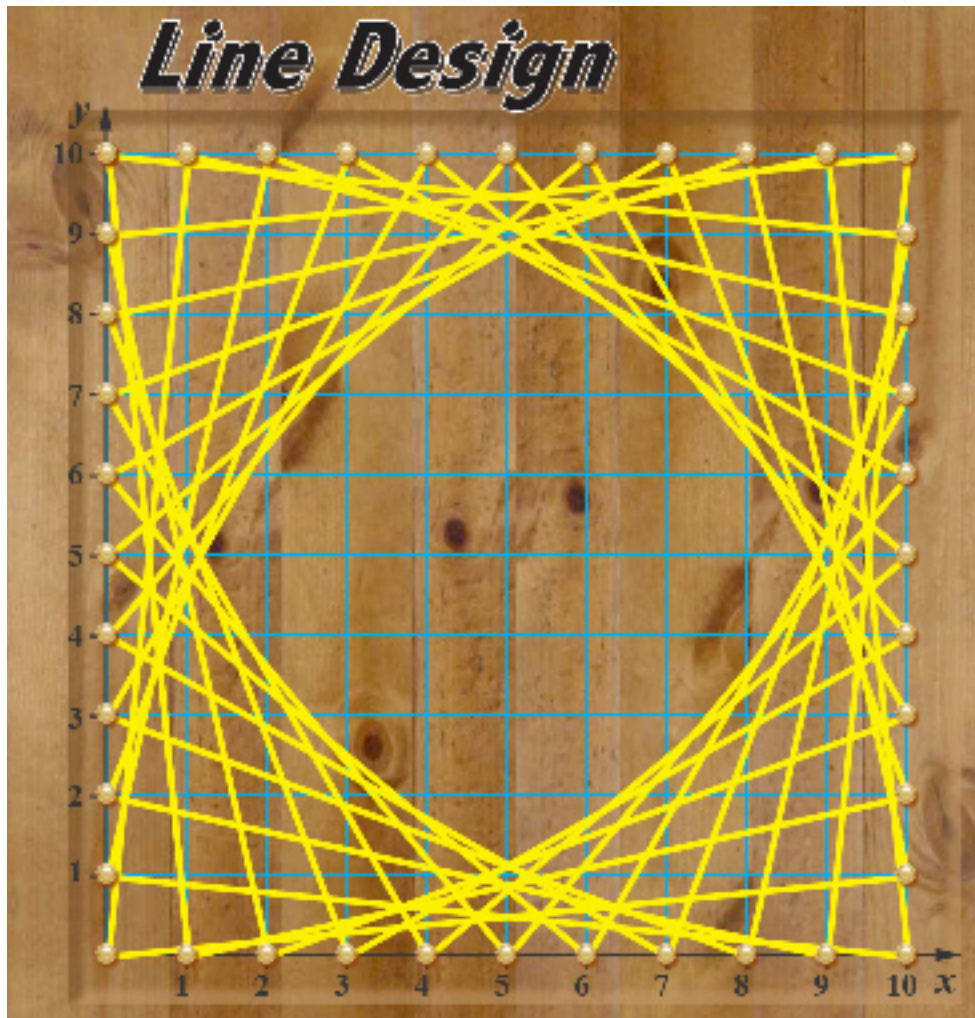
How many line segments were used in your design?



## The Square Pattern

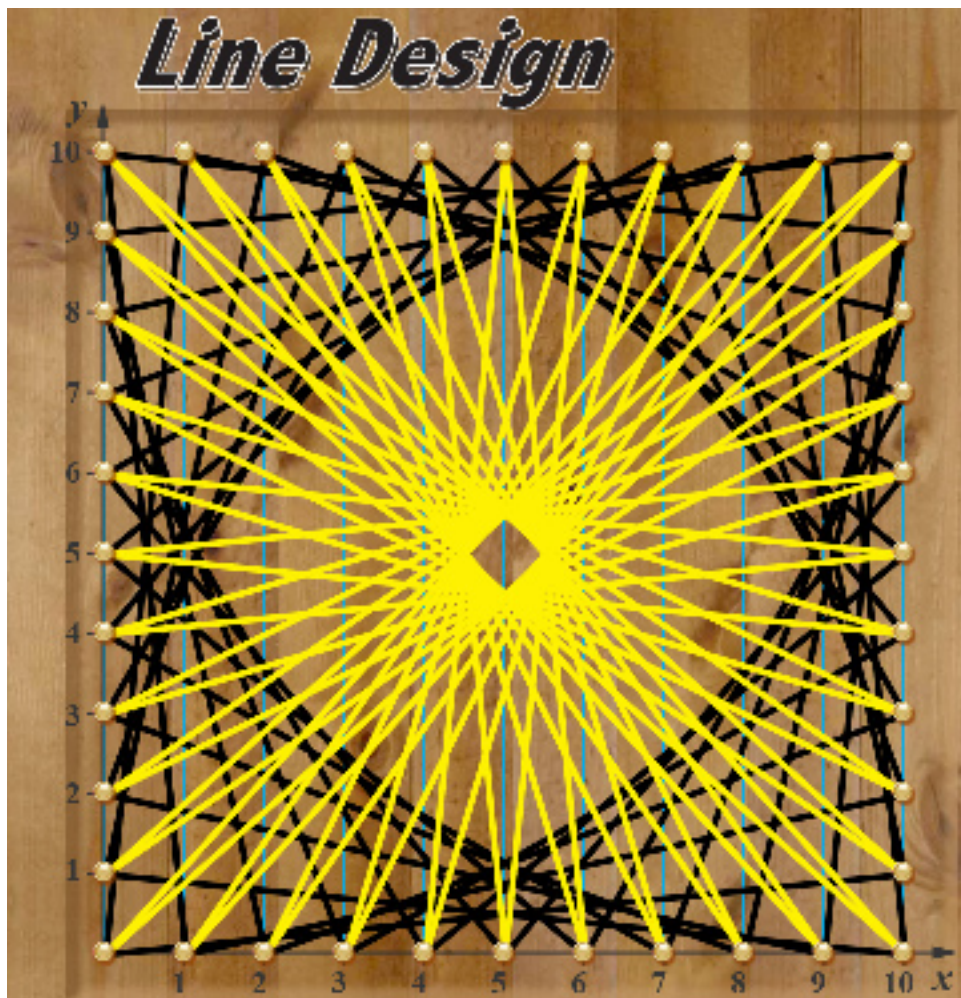
*Curious Fact: This design is made up of only straight lines, yet it appears to be curved.*

Tap the dog ear icon on the right corner of the note pad to change the design pattern to the square pattern. Label the x and y axes to be able to locate the end points of the first line segment. Draw a yellow line from (0,10) to (1,0). Tap Go! to replicate the line segment and produce this design.



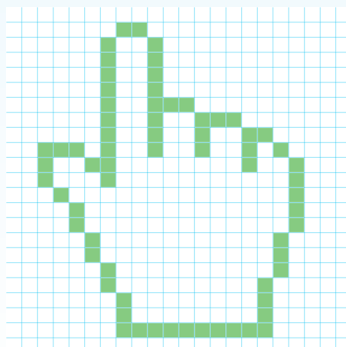
## Getting Creative with the Square Pattern

Tap the dog ear icon on the right corner of the note pad to change the design pattern to a square. Label the x and y axes to be able to locate the endpoints of the line segments used to create this design.



1. Using black and draw a line segment from  $(0,10)$  to  $(1,0)$ . Tap Go! to replicate the line segments.
2. Tap yellow and draw a line segment from  $(0,10)$  to  $(9,0)$ . Tap Go! to replicate the line segments.

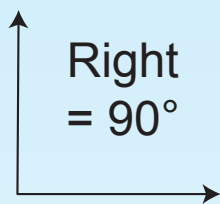
Experiment with your own patterns using two colors to make each design.



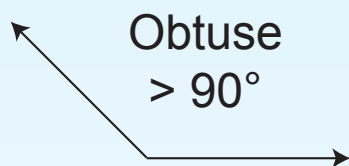


## A Design Based on the Square

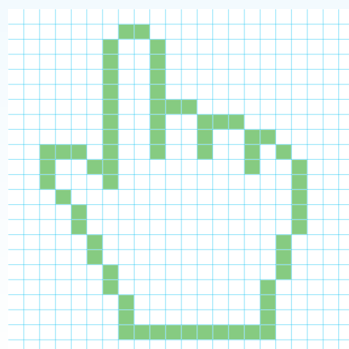
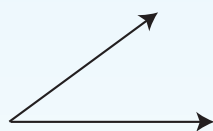
*All four of the angles in a square are right angles.*



*If a parallelogram is not a rectangle then 2 angles are acute angles and 2 angles are obtuse angles.*



Acute  
<  $90^\circ$

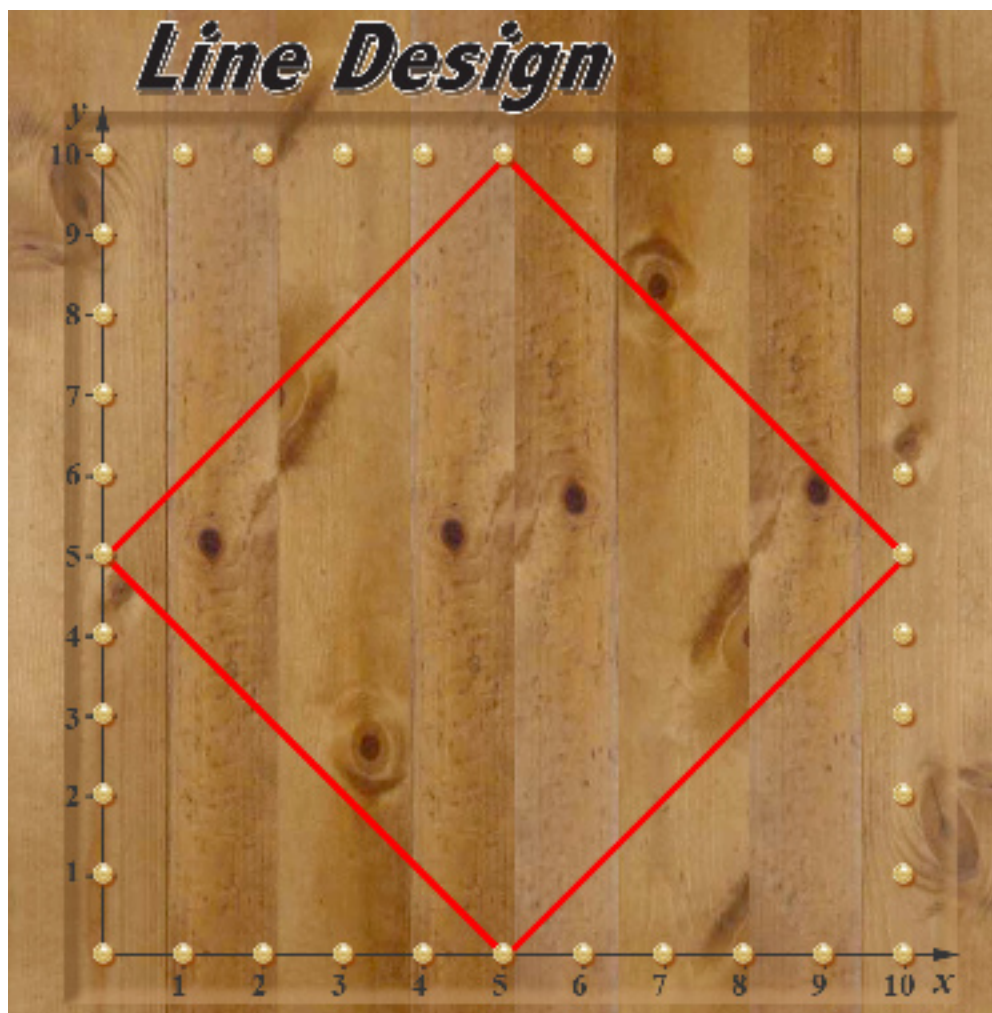
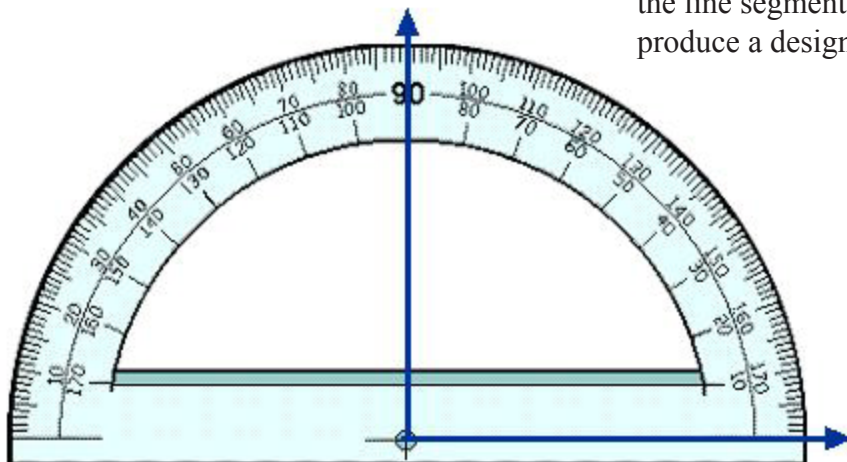


Draw a square by using this coordinate pairs as the endpoints for 4 line segments. Each angle is  $90^\circ$ . Tap Go! to replicate the design.

| Segment | End Points      |
|---------|-----------------|
| 1       | (0,5) - (5,0)   |
| 2       | (5,0) - (10,5)  |
| 3       | (10,5) - (5,10) |
| 4       | (5,10) - (0,5)  |

**GO!**

Tap Go! to replicate the line segment and produce a design.



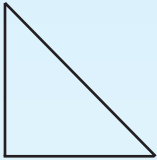


## Classifying Triangles

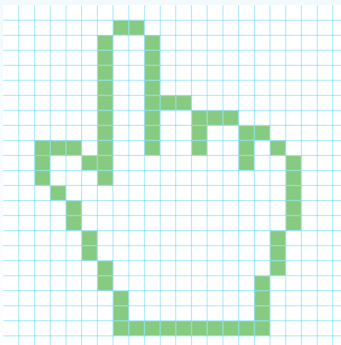
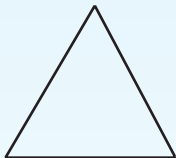
Obtuse  
 $> 90^\circ$



Right  
 $= 90^\circ$

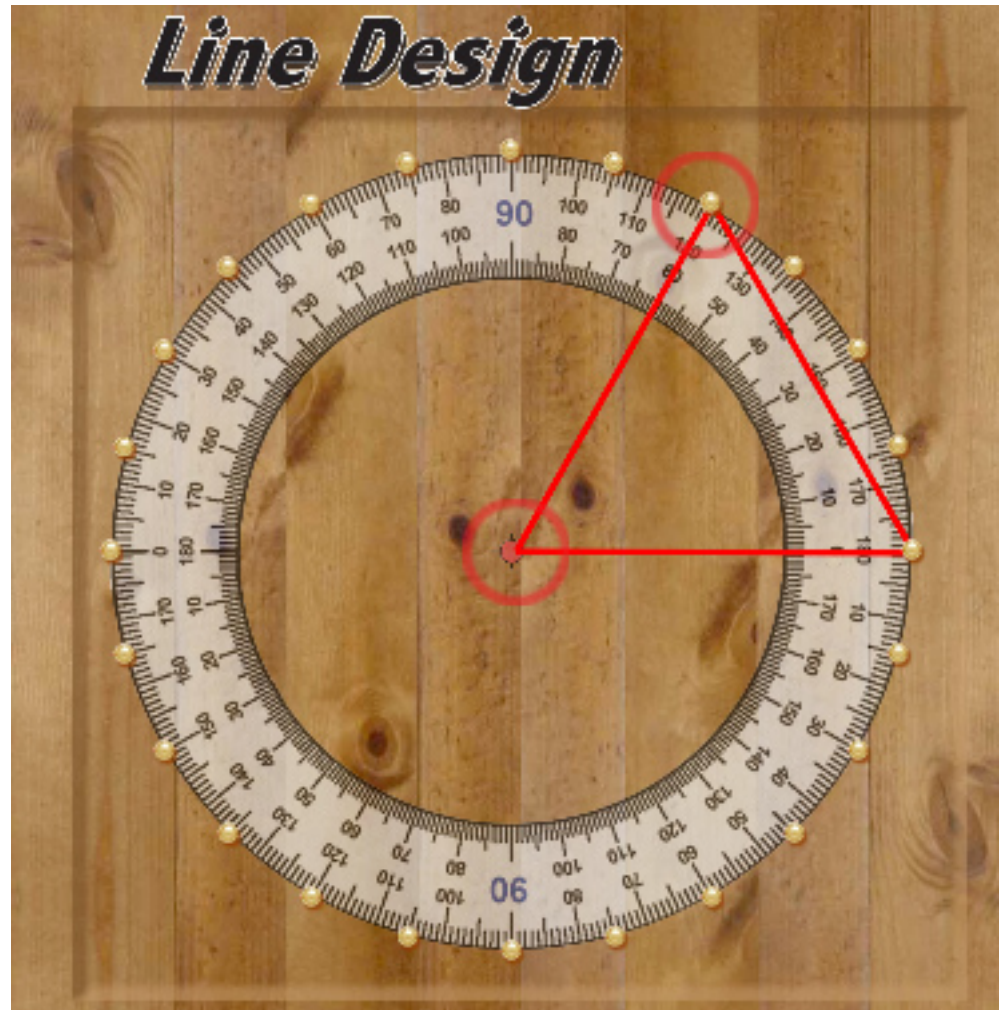


Acute  
 $< 90^\circ$



Triangles can be classified by the sizes of the angles. Triangles with an angle that is equal to  $90^\circ$  are **right triangles**, triangles with an angle greater than  $90^\circ$  are **obtuse triangles** and triangles with all three angles less than  $90^\circ$  are **acute triangles**.

Use the circle pattern for this exploration. Tap the protractor icon to display a protractor on the Line Design Playground.



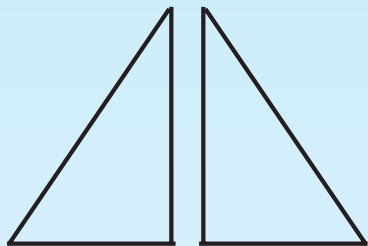
Draw a triangle by dragging from the center of the circle to the  $0^\circ$  mark on the right. Draw a second line segment from that point to the  $60^\circ$  mark and then back to the center.

Which type of triangle is this?

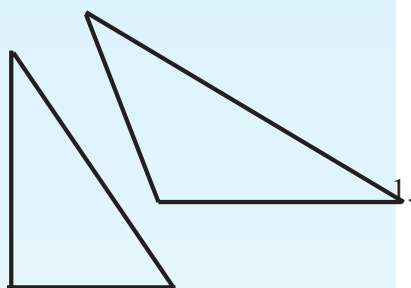
- A. obtuse
- B. right
- C. acute

Use the Line Design Playground to draw each type of triangle: obtuse, right and acute.

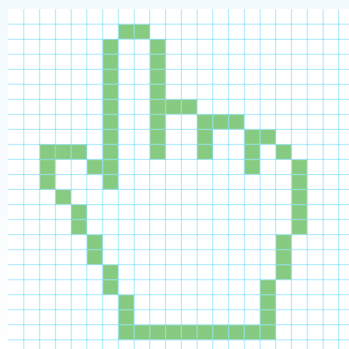
## Congruent Triangles



Congruent Triangles



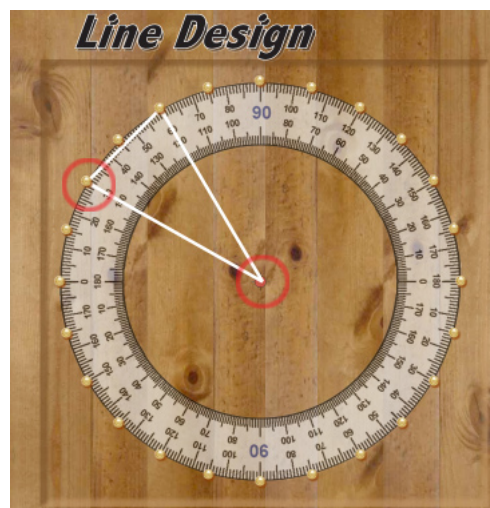
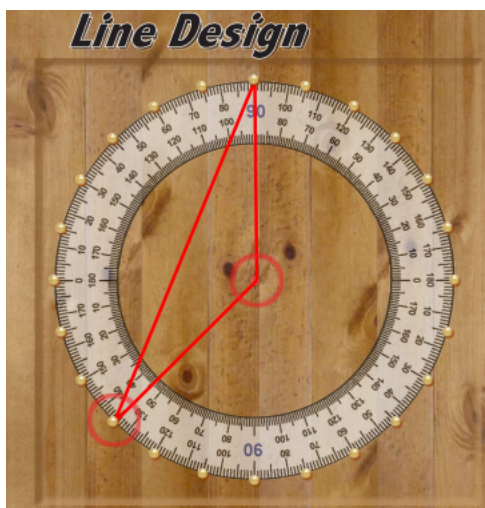
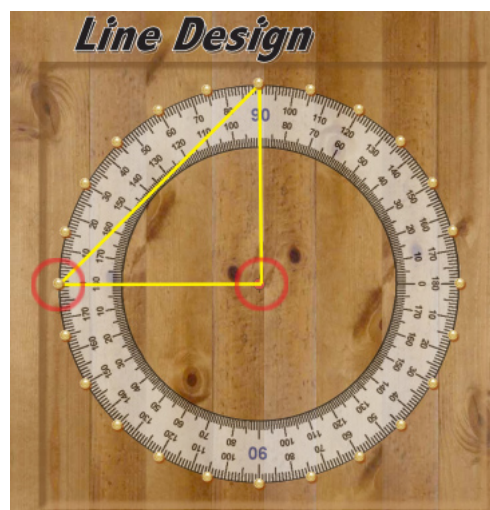
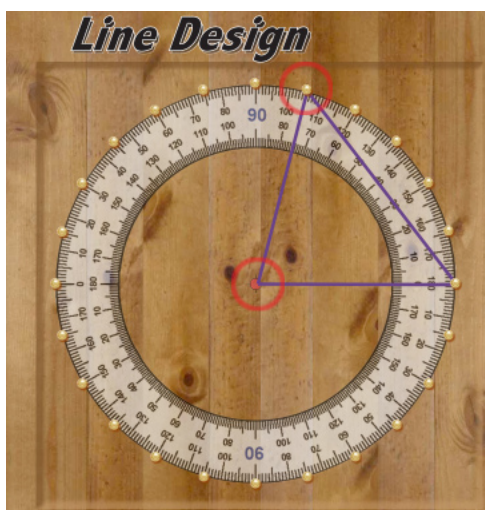
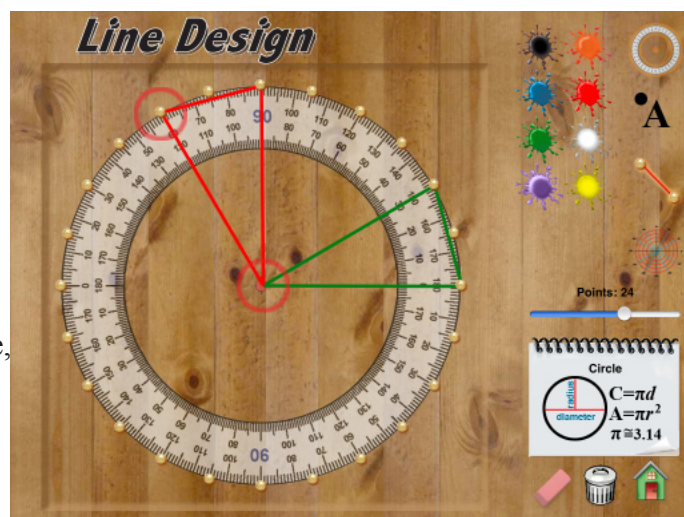
Not Congruent



Triangles are **congruent** if all **corresponding sides** and **interior angles** are congruent. **Congruent** triangles have the same shape and size.

Set the circle pattern for 24 points on the circle.

For each example, copy the triangle to the Line Design Playground and then draw a congruent triangle using a different color. Here, for example, the red and green triangles are congruent.



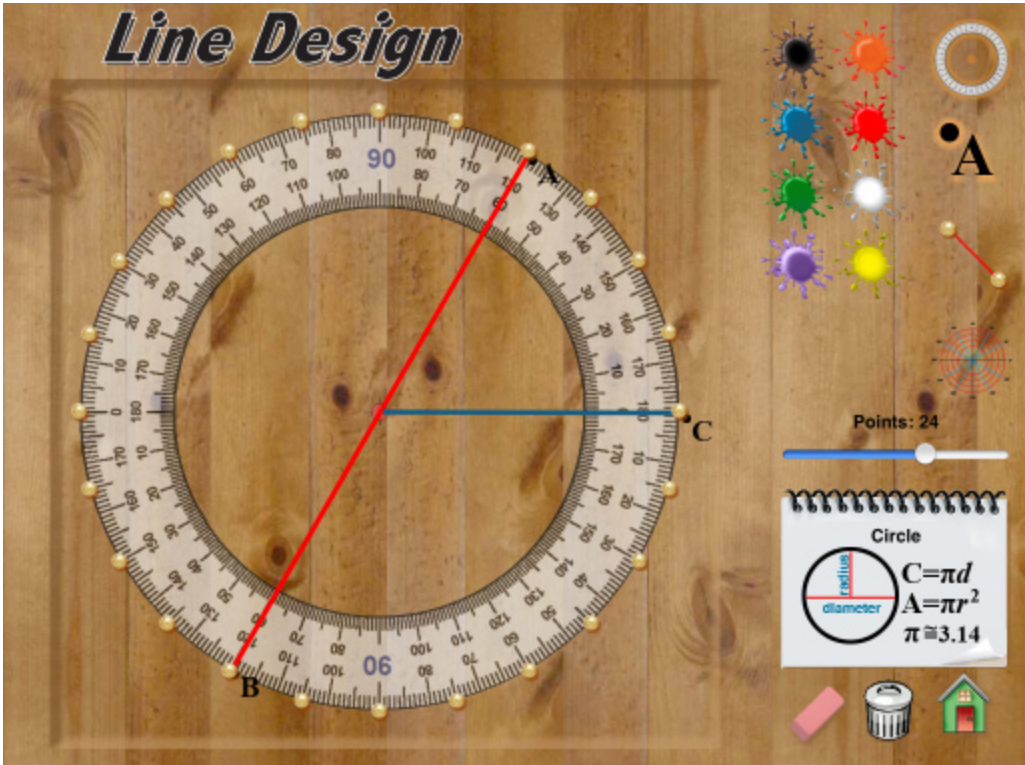


# Parts of a Circle

The diameter of any circle is twice the radius.

The distance from the center of the circle to edge is the *radius*. The line from any two points on the edge that passes through the center is the *diameter*. The edge or boundry of a circle is the *circumference*.

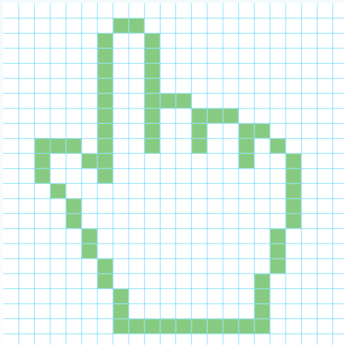
- Copy this diagram on the Line Design Playground. Write either diameter or radius to name the part shown for each color.



Red \_\_\_\_\_  
 Blue \_\_\_\_\_

Complete this table by writing the measurement of the diameter or radius for each circle.

| Radius | Diameter |
|--------|----------|
| 5      |          |
|        | 8        |
| 3      |          |
|        | 12       |
| 9      |          |

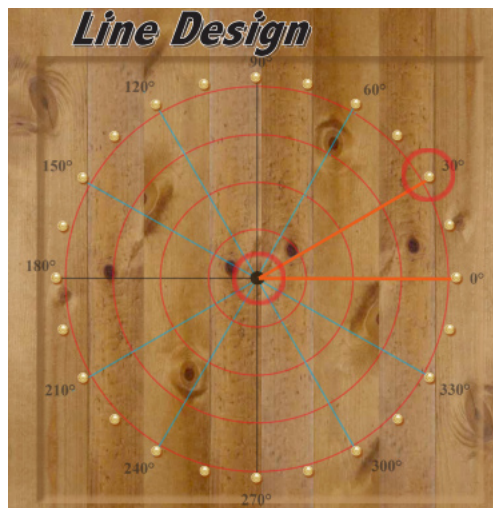


# Central Angles and Inscribed Angles

In math, a *conjecture* is a statement that is believed to be true but not yet proved.

A *central angle* is formed when two radii form an angle with its vertex at the center of a circle.

An *inscribed angle* is formed by two *chords* in a circle which have a common endpoint. The common endpoint forms the *vertex* of the inscribed angle.



Use the Line Design Playground for this project. Set the number of points to 24. Draw a central angle with a measurement of 30° as shown.

The *Inscribed Angle Conjecture* describes the relationship between an inscribed angle and a central angle with the same endpoints. It states that measure of the central angle is twice the measure of the inscribed angle.

For this 30° angle, draw an inscribed angle on the Line Design Playground.

1. Draw a central angle that measures 60°. Draw a corresponding inscribed angle. What is the measure of the inscribed angle?

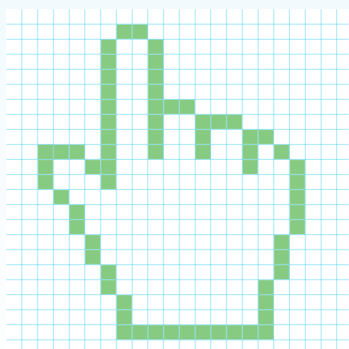
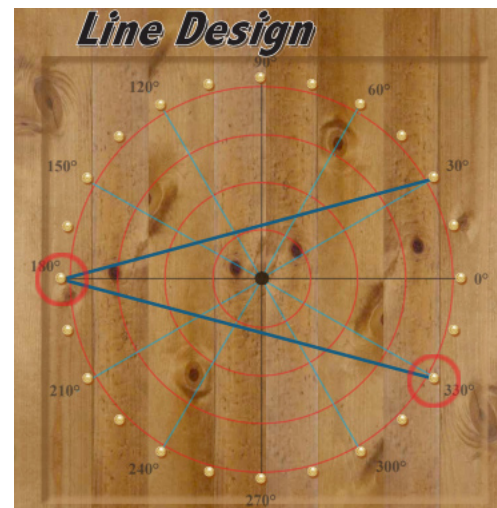
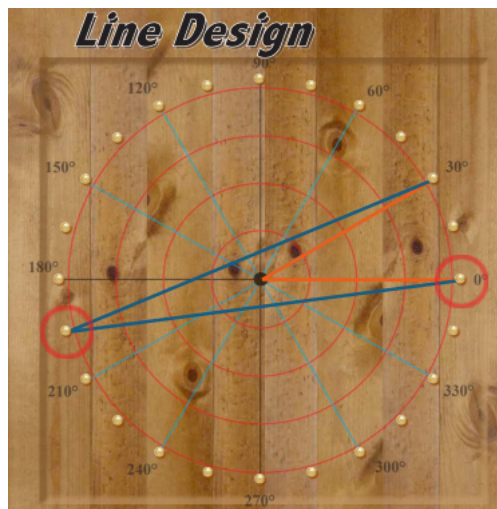
---

2. Draw a inscribed angle that measures 120°. What is the measure of this angle?

---

3. Draw the central angle that corresponds to the inscribed angle shown here. What is the measure of the central angle?

---

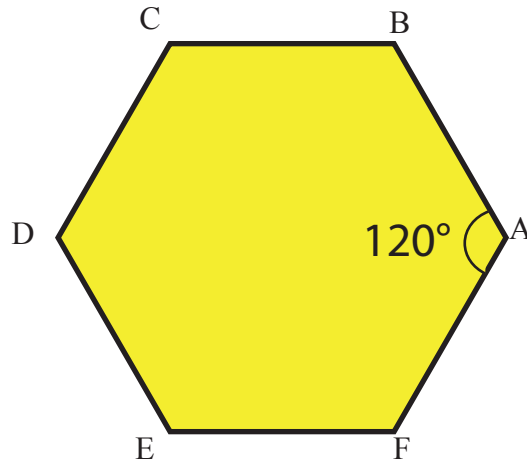


# Polygons



Select the point labelling tool, then tap the vertices of a figure to label the points.

The *interior angles* of a polygon are the angles formed at each vertex on the inside of the polygon. If a polygon has  $n$  sides, then there are  $n$  vertices and  $n$  interior angles.



The sum of the interior angles of a polygon can be found using this formula:

$$S = 180(n - 2)$$

For a regular hexagon each interior angle is  $120^\circ$ , therefore, the sum of the interior angles is  $720^\circ$ .

Use the Line Design Playground for this activity. Set the number of points to 24. Tap the degrees icon to label the degrees.



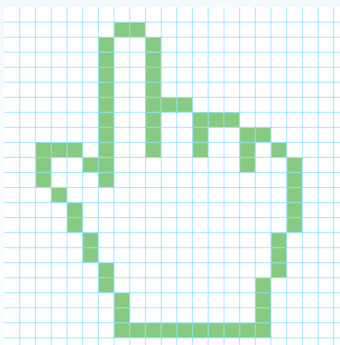
Draw a line segment from the  $0^\circ$  marker to the  $60^\circ$  marker. Choose Go! to replicate the line segment. Label the vertices A through E.

Which figure will result if you repeat this procedure but draw a line segment from the  $0^\circ$  marker to the  $90^\circ$  marker? Try it.

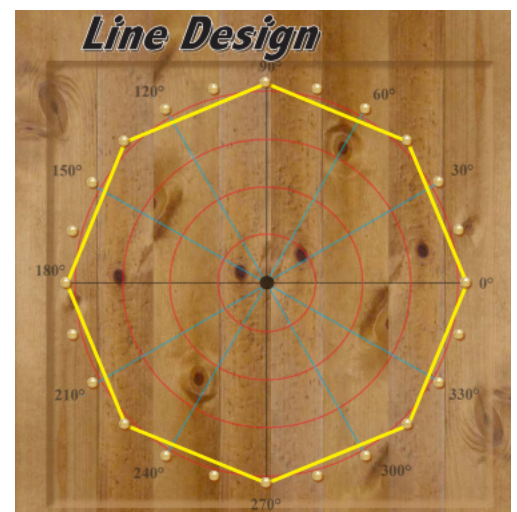
\_\_\_\_\_

This diagram begins with a line segment from  $0^\circ$  to  $45^\circ$ . It divides the circle into 8 equal parts and produces a regular octagon.

Find the sum of the interior angles for a triangle, square, pentagon, hexagons, octagon and decagon. Complete this chart.

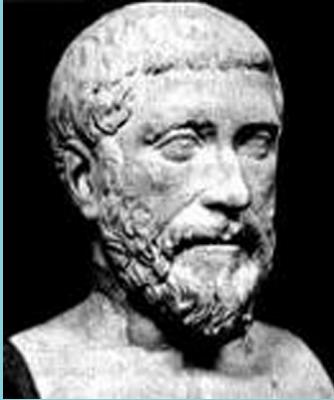


| Sides | Sum of Interior Angles |
|-------|------------------------|
| 3     | $180^\circ$            |
| 4     |                        |
| 5     |                        |
| 6     | $720^\circ$            |
| 8     |                        |
| 10    |                        |



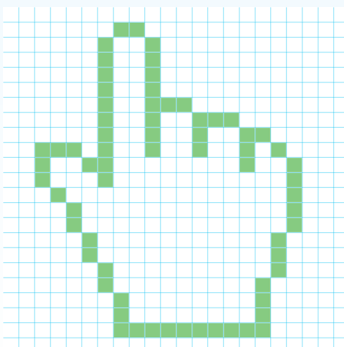


# The Pythagoreans and their Secret Symbol

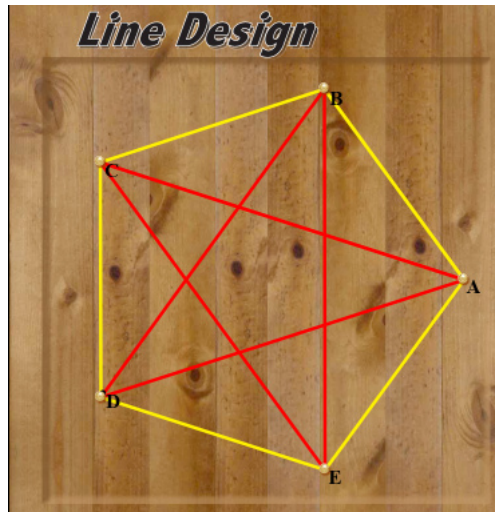


**Pythagoras**

*Pythagoras was a Greek mathematician and philosopher. He is credited with discovering the relationships of the sides of a right triangle which today is known as the Pythagorean theorem.*



The followers of Pythagoras known as the Pythagoreans believed that the pentagram embodied mathematical perfection. Construct a pentagram by setting the number of points on the circle pattern to 5.



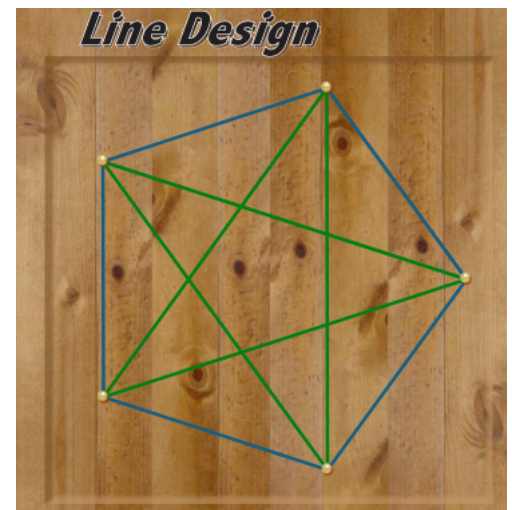
The points of the pentagram were thought to represent the five planets Jupiter, Mercury, Mars, Saturn, and Venus. Label the points A through E.

Ancient Chinese philosophy held that the points of the pentagram represented five phases or elements:

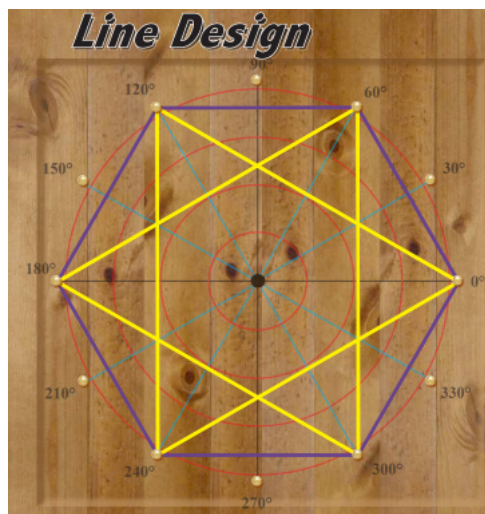
- A. Fire
- B. Wood
- C. Water
- D. Metal
- E. Earth

The Pythagoreans used the pentagram as a secret symbol. To attend special gatherings where mathematics and philosophy would be discussed a Greek had to show a pentagram to gain admission.

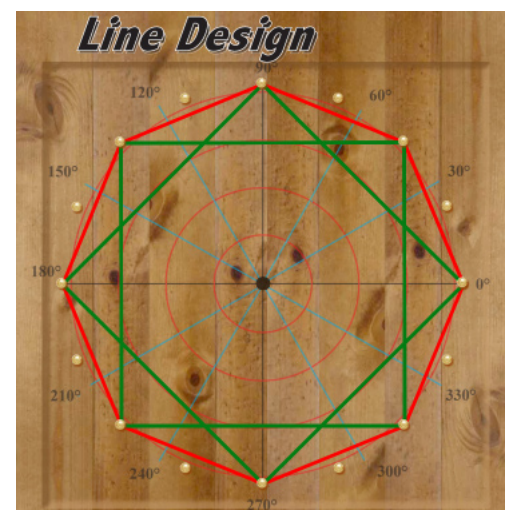
Draw a blue pentagon with an inscribed green pentagram.



Set the number of points to 12. Draw hexagon with an inscribed hexagram.

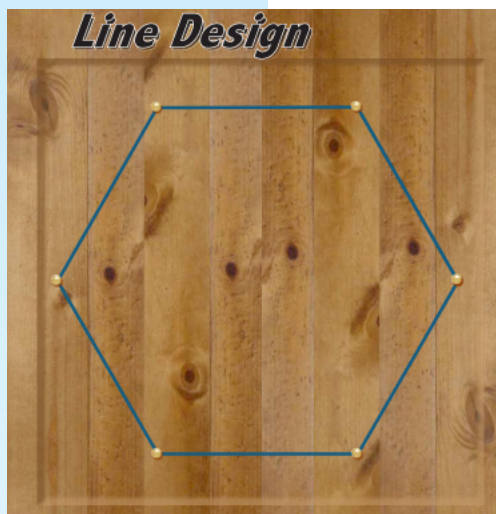


Set the number of points to 16. Draw an octagon with an inscribed octagram.



# Identifying and Naming Polygons

When the circle pattern is set to a specific number of points and adjacent points are connected a regular polygon is the result. For example, if the number of points is 6 and adjacent points are connected the design is a hexagon. Each point is a vertex and since all the line segments are the same length, the figure is a regular polygon.



Complete this table by writing the name for each polygon. Make each figure on the Line Design Playground.

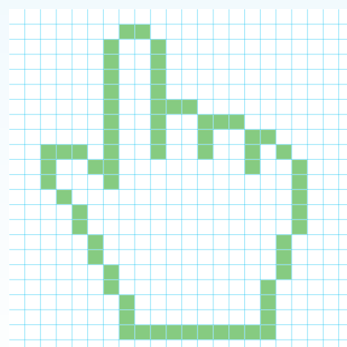
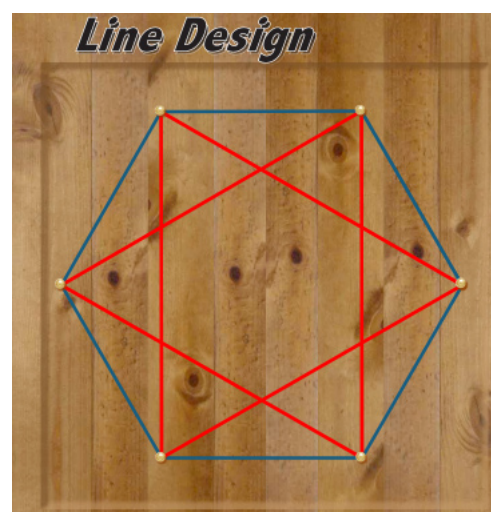
| Vertices | Polygon   |
|----------|-----------|
| 4        | Square    |
| 5        | Pentagon  |
| 6        | Hexagon   |
| 7        |           |
| 8        |           |
| 9        |           |
| 10       |           |
| 11       | undecagon |
| 12       |           |

## Greek Prefixes

5 Penta-  
6 Hexa-  
7 Septa-  
8 Octo-  
9 Nona-  
10 Deca-  
11 Undeca-  
12 Dodeca-

The corresponding star-like shapes also have special names, for example, pentagon and pentagram. Use the Line Design Playground to make a star-like design for each regular polygon. Write the name of the star-like shapes.

| Polygon | Star      |
|---------|-----------|
| 5       | Pentagram |
| 6       | Hexagram  |
| 7       |           |
| 8       |           |
| 9       |           |
| 10      |           |
| 11      |           |
| 12      |           |





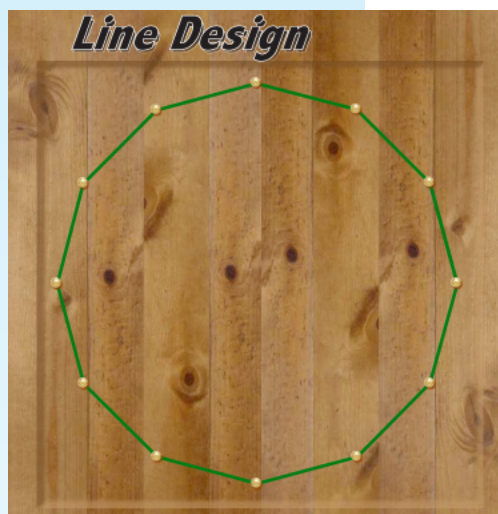
## Designs Based on a Circle

The Greeks were the first known culture to divide a circle into 360 degrees. Perhaps 360 was selected because it is approximately the number of days that it takes the earth to orbit the sun. Fortunately, 360 is a very good number. It has 23 factors: 360, 180, 120, 90, 72, 60, 45, 40, 36, 30, 24, 20, 18, 12, 10, 9, 8, 6, 5, 4, 3, and 2.



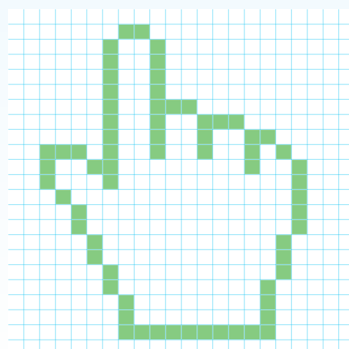
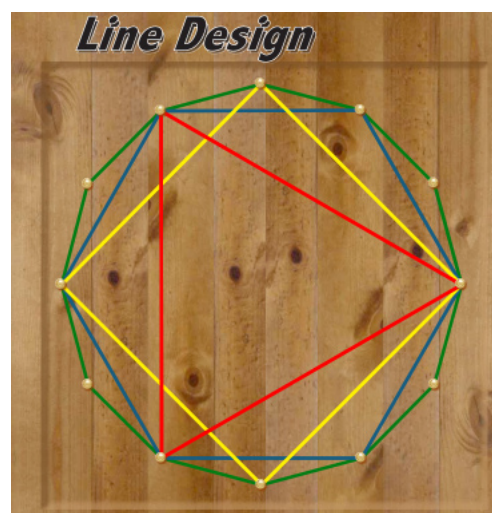
The circle design pattern can be set to any number between 3 and 36, but factors of 360 are particularly good choices for designs.

*For the experiments on this page set the number of points to 12.*



1. Select green and draw a line segment from one adjacent point to another. Tap the Go! button. The result is a 12-sided figure called a dodecagon.
2. Next select blue and draw a line segment from any point to another point but skip one point. Tap Go! Because we connected every other point, the result is a 6-sided figure or hexagon.
3. Select yellow and draw a line segment skipping 2 points. This time when you tap Go! a square is the result. Your Line Design Playground should now show three regular polygons.

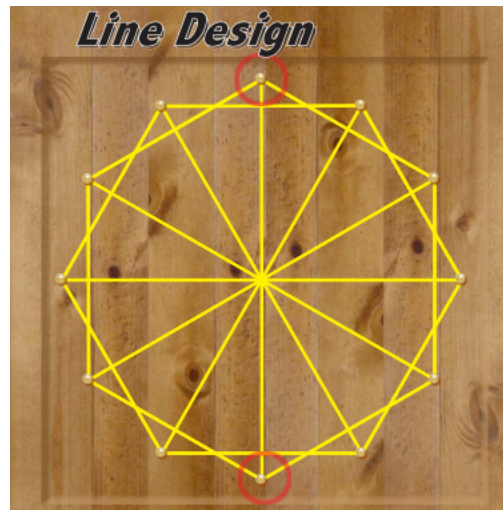
4. Finish this design by selecting red and drawing a line segment from any point to another point skipping 3 points. Tap Go! and an equilateral triangle is the result.
5. Set the number of points to a different factor of 360. Try this experiment again to produce a different Line Design.



# Hidden Figures

For this experiment set the number of points to 12. Select yellow and draw a line segment to connect every other point. Repeat this procedure to draw two hexagons. Complete the drawing by figure by drawing 6 diagonals through the center point.

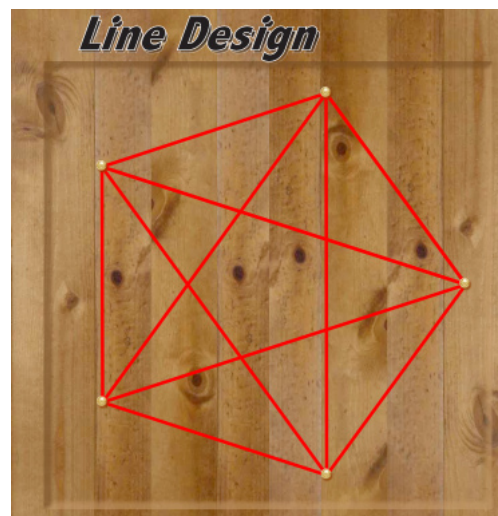
How many hidden cubes can you find in this figure.



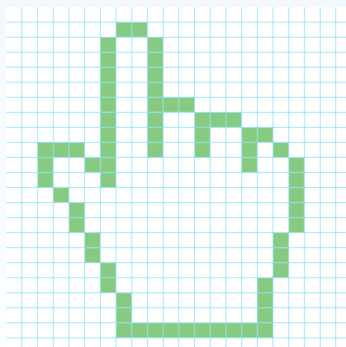
Set the Line Design Playground to 12 points. Draw this figure. How many hidden triangles can you find in this design?



Set the Line Design Playground to 5 points. Draw this figure. How many hidden triangles can you find in this design?



Set the Line Design Playground to 8 points. Draw this figure. How many hidden triangles can you find in this design?



# Hands-On Math: Line Design

Ventura Educational Systems

